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Innovative Applications of O.R.

Efficiency measurement for parallel production systems

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ABSTRACT

In the real world there are systems which are composed of independent production units. The conventional data envelopment analysis (DEA) model uses the sum of the respective inputs and outputs of all component units of a system to calculate its efficiency. This paper develops a parallel DEA model which takes the operation of individual components into account in calculating the efficiency of the system. A property owned by this parallel model is that the inefficiency slack of the system can be decomposed into the inefficiency slacks of its component units. This helps the decision maker identify inefficient components and make subsequent improvements. Another property is that the efficiency calculated from this model is smaller than that calculated from the conventional DEA model. Few systems will have perfect efficiency score; consequently, a stronger discrimination power is gained. In addition to theoretical derivations, a case of the national forests of Taiwan is used as an example to illustrate the whole idea.

1. Introduction

Performance evaluation is an important task for a decision making unit (DMU) to find its weaknesses so that subsequent improvements can be made. Since the pioneering work of Charnes et al. (1978), data envelopment analysis (DEA) has demonstrated to be an effective technique for measuring the relative efficiency of a set of DMUs which utilize the same inputs to produce the same outputs. Suppose there are n DMUs. The kth DMU utilizes m inputs X_{ik}, $i = 1,\ldots,m$ to produce s outputs Y_{rk}, $r = 1,\ldots,s$. Its efficiency E_k is calculated via the following CCR model (Charnes et al., 1978):

$$\begin{aligned} E_k = \max.\quad & \sum_{r=1}^{s} u_r Y_{rk} \\ \text{s.t.}\quad & \sum_{i=1}^{m} v_i X_{ik} = 1, \\ & \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0, \quad j = 1,\ldots,\ n \\ & u_r, v_i \geqslant \varepsilon, \quad r = 1,\ldots,s,\ \ i = 1,\ldots,m, \end{aligned} \tag{1}$$

where u_r and v_i are the most favorable multipliers to be applied to the rth output and ith input for DMU k in calculating its efficiency E_k and ε is a small non-Archimedean quantity (Charnes et al., 1979; Charnes and Cooper, 1984) which prohibits any input/output factor to be ignored.

In the real world there are cases that a DMU is composed of a set of components, and each utilizes the same inputs to produce the same outputs. A typical example is a firm with several plants, each operates independently. Each of the firm's inputs and outputs is the sum of those of all its plants. The general case is a parallel production system k with q production units, as depicted in Fig. 1, where each production unit p, $p = 1,\ldots,q$ converts inputs X_{ik}^{p}, $i = 1,\ldots,m$ into outputs Y_{rk}^{p}, $r = 1,\ldots,s$, independently. The sums of all X_{ik}^{p} over p, $\sum_{p=1}^{q} X_{ik}^{p}$, and all Y_{rk}^{p} over p, $\sum_{p=1}^{q} Y_{rk}^{p}$, are the input X_{ik} and output Y_{rk} of the system, respectively. Castelli et al. (2004) present hierarchical structures where each DMU is composed of consecutive stages of parallel subunits and Färe and Grosskopf (2000) propose a network model for measuring the efficiency of the system. However, the operation of each component of the system is treated independently, without considering the relationship among the components.

According to Koopmans (1977), the feasible set in the space of commodity flows for a firm is the convex hull of the set of vectors resulted from consolidating all combinations of its production units. Based on this idea, Färe and Primont (1984) develop a DEA model to measure the efficiency of multi-plant firms. Kao (1998) applies the same idea to measure the efficiency of forest districts with multiple working circles. Conceptually, the efficiency boundary of the convex hull is the short-run production frontier of the firm. The frontier super-imposed upon all short-run production frontiers is the long-run production frontier for all production units. Based on this idea, Kao (2000) develops a DEA model to measure the short-run and long-run efficiencies of every production unit.

In this paper we will investigate the production system with parallel production units. A parallel DEA model is developed to calculate the efficiency of the whole system as well as the efficiencies of individual production units. Based on the results, the decision

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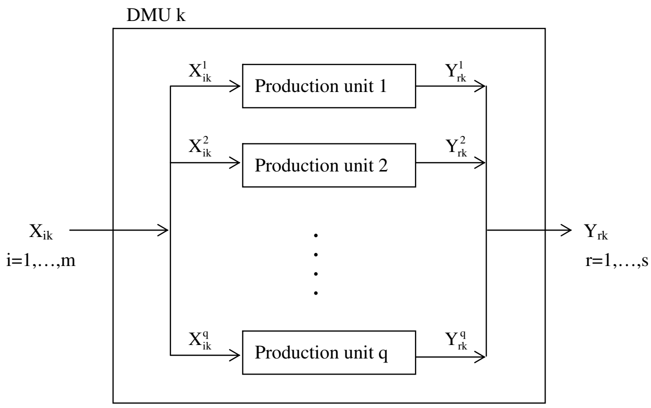

Fig. 1. The parallel production system, where a DMU k has q production units each utilizing the same m inputs to produce the same s outputs.

maker is able to reallocate resources to different production units in the system more efficiently so that the efficiency of the system can be improved.

The structure of this paper is organized as follows. Firstly, a parallel DEA model is developed to calculate the system and component efficiencies. An example of single-input and single-output, which can be illustrated graphically, is designed for better explanation. Then a real world example of forest production in Taiwan is used to illustrate the whole idea. Finally, some conclusions are drawn from the discussion.

2. The parallel model

The conventional DEA Model (1) measures the performance of a DMU in terms of efficiency. As a matter of fact, the performance can also be measured from the viewpoint of inefficiency, since inefficiency 1-E_k is the complement of efficiency E_k. The objective of maximizing efficiency is equivalent to minimizing inefficiency. Following Model (1), the inefficiency of DMU k is 1-$\sum_{r=1}^{s}u_rY_{rk}$, which is equal to the slack s_k in $\sum_{r=1}^{s}u_rY_{rk}-\sum_{i=1}^{m}v_iX_{ik}+s_k=0$. Note that $\sum_{i=1}^{m}v_iX_{ik}$ is equal to 1. Model (1) is equivalent to the following program:

$$\begin{aligned}
\min.\quad & s_k,\\
\text{s.t.}\quad & \sum_{i=1}^{m}v_iX_{ik}=1,\\
& \sum_{r=1}^{s}u_rY_{rk}-\sum_{i=1}^{m}v_iX_{ik}+s_k=0,\\
& \sum_{r=1}^{s}u_rY_{rj}-\sum_{i=1}^{m}v_iX_{ij}\leqslant 0,\quad j=1,\ldots,n,\ j\neq k,\\
& u_r,v_i\geqslant\varepsilon,\quad r=1,\ldots,s,\ i=1,\ldots,m.
\end{aligned}\tag{2}$$

Here the inefficiency score is represented by the slack variable s_k.

Next, in the parallel production system, each input/output of the system is the sum of those of all its production units. Hence, we have

$$\begin{aligned}
&\sum_{r=1}^{s}u_rY_{rk}-\sum_{i=1}^{m}v_iX_{ik}+s_k,\\
&\quad=\sum_{r=1}^{s}u_r(Y_{rk}^1+Y_{rk}^2+\cdots+Y_{rk}^q)-\sum_{i=1}^{m}v_i(X_{ik}^1+X_{ik}^2+\cdots+X_{ik}^q)+s_k,\\
&\quad=\sum_{p=1}^{q}\left(\sum_{r=1}^{s}u_rY_{rk}^p-\sum_{i=1}^{m}v_iX_{ik}^p\right)+s_k=0,
\end{aligned}\tag{3}$$

where X_{ik}^p and Y_{rk}^p are the ith input and rth output, respectively, of the pth production unit within DMU k, and there are q production units in this DMU. Note that $(\sum_{r=1}^{s}u_rY_{rk}^p-\sum_{i=1}^{m}v_iX_{ik}^p)$ in the last equation represents the production mechanism of the pth production unit. It must be non-positive in determining the most favorable multipliers u_r and v_i for DMU k to fulfill the definition of efficiency. Let s_k^p denote the slack associated with the pth production unit. The total slack of the system, s_k, can be allocated to its q production units: $s_k=\sum_{p=1}^{q}s_k^p$. Thus, the last equation of (3) becomes

$$\sum_{p=1}^{q}\left(\sum_{r=1}^{s}u_rY_{rk}^p-\sum_{i=1}^{m}v_iX_{ik}^p+s_k^p\right)=0.\tag{4}$$

Since each quantity in the parentheses is equal to zero, we have derived a set of q constraints:

$$\sum_{r=1}^{s}u_rY_{rk}^p-\sum_{i=1}^{m}v_iX_{ik}^p+s_k^p=0,\quad p=1,\ldots,q.\tag{5}$$

By the same token, the constraint associated with each DMU other than k in Model (2) is replaced by the same constraints corresponding to its q production units. Certainly, each DMU can have a different number of production units. Here we use a common number q just for simplification of notation. For more general cases where the jth DMU has q_j production units, one just replaces q by q_j accordingly.

In sum, the DEA model for calculating the relative inefficiency of a set of n DMUs, each has q parallel production units, is

$$\begin{aligned}
\min.\quad & \sum_{p=1}^{q}s_k^p,\\
\text{s.t.}\quad & \sum_{i=1}^{m}v_iX_{ik}=1,\\
& \sum_{r=1}^{s}u_rY_{rk}^p-\sum_{i=1}^{m}v_iX_{ik}^p+s_k^p=0,\quad p=1,\ldots,q,\\
& \sum_{r=1}^{s}u_rY_{rj}^p-\sum_{i=1}^{m}v_iX_{ij}^p\leqslant 0,\quad p=1,\ldots,q,\ j=1,\ldots,n,\ j\neq k,\\
& u_r,v_i\geqslant\varepsilon,\quad r=1,\ldots,s,\ i=1,\ldots,m.
\end{aligned}\tag{6}$$

This model will be enumerated for n times, once for each DMU, to calculate the inefficiency slacks of the systems as well as their subordinated production units. From the inefficiency decomposition, the decision maker is able to identify the production units with large inefficiency slacks and make subsequent improvements.

At this point, it must be noted that the efficiency score of the wth production unit of the kth DMU is not 1-s_k^w. The reason is because $\sum_{i=1}^{m}v_iX_{ik}^w$ is not equal to 1. According to the second constraint of Model (6), s_k^w must be divided by $\sum_{i=1}^{m}v_iX_{ik}^w$ to obtain the efficiency score of $1-s_k^w/\sum_{i=1}^{m}v_iX_{ik}^w$.

The basic difference between this parallel DEA model and the conventional DEA model is that the constraint for each DMU has been replaced by those associated with its subordinated production units. Referring to (4), the sum of the constraints associated with the production units is equal to the constraint of the system. In other words, constraints in Model (6) are stronger than those of Model (2). Consequently, the efficiency score calculated from the parallel DEA model is smaller than that calculated from the conventional DEA model.

In the single-stage hierarchical model of Castelli et al. (2004) and the network model of Färe and Grosskopf (2000), each production unit has different set of multipliers while production units within the same DMU in this study have the same set of multipliers. This is how production units within the same DMU are related. If the objective function of Model (6) is replaced by s_k^w and the first constraint is replaced by $\sum_{i=1}^{m}v_iX_{ik}^w=1$, then we have the conventional CCR model for measuring the efficiency of the wth production unit of the kth DMU.

Model (6) has a dual of the following form:

$$\begin{aligned}
\max.\quad & -\mu_0+\varepsilon\left(\sum_{r=1}^{s}u_r+\sum_{i=1}^{m}v_i\right)\\
\text{s.t.}\quad & -\sum_{j=1}^{n}\sum_{p=1}^{q}\mu_j^pY_{rj}^p+s_r^-=0,\quad r=1,\ldots,s,\\
& -\mu_0X_{ik}+\sum_{j=1}^{n}\sum_{p=1}^{q}\mu_j^pX_{ij}^p+s_i^+=0,\quad i=1,\ldots,m,\\
& -\mu_k^p\leqslant 1,\quad p=1,\ldots,q,\\
& \mu_0,\mu_k^p,\quad p=1,\ldots,q\quad \text{unconstrained in sign.}
\end{aligned}\tag{7}$$

Since $\sum_{p=1}^{q}X_{ij}^p=X_{ij}$ and $\sum_{p=1}^{q}Y_{rj}^p=Y_{rj}$, by setting $\lambda_k^p=\mu_k^p+1$ and $\sigma=\mu_0+1$, Model (7) can be reformulated as

$$\begin{aligned}
\max.\quad & 1-\sigma+\varepsilon\left(\sum_{r=1}^{s}u_r+\sum_{i=1}^{m}v_i\right)\\
& \sum_{j=1}^{n}\sum_{p=1}^{q}\lambda_j^pY_{rj}^p-s_r^-=Y_{rk},\quad r=1,\ldots,s,\\
& \sum_{j=1}^{n}\sum_{p=1}^{q}\lambda_j^pX_{ij}^p+s_i^+=\sigma X_{ik},\quad i=1,\ldots,m,\\
& \lambda_j^p\geqslant 0,\quad p=1,\ldots,q,\ j=1,\ldots,n.
\end{aligned}\tag{8}$$

The objective value is the inefficiency of DMU k. Complementarily, its efficiency score is $\sigma-\varepsilon(\sum_{r=1}^{s}u_r+\sum_{i=1}^{m}v_i)$, an expression used in the conventional DEA model.

In this dual formulation, if all λ_j^p, $p=1,\ldots,q$ associated with the production units within the same DMU are the same, then Model (8) boils down to the conventional CCR dual model, since $\sum_{p=1}^{q}X_{ij}^p=X_{ij}$ and $\sum_{p=1}^{q}Y_{rj}^p=Y_{rj}$. This implies that the parallel model of this paper additionally takes the production units within a DMU into account in measuring the system's efficiency.

3. Graphical illustration

Consider a simple case of two DMUs, A and B, using one input X to produce one output Y. DMU A has two production units, a_1 and a_2, and DMU B has three production units b_1, b_2, and b_3. Their input and output data are shown in Table 1. As depicted in Fig. 2, if the CCR model is applied to measure the efficiencies of A and B directly, then the production frontier is the straight line OQ, which passes through the origin and DMU A. The associated efficiency scores for A and B are 1 and 25/36, respectively, where A is efficient. However, if the efficiencies of A and B are measured indirectly via their production units by applying Model (6), then the production frontier is line OP, which passes through the origin and production unit a_1. In this case, the efficiency scores of A and B are 9/10 and 10/16, respectively, as shown in the last column of Table 1, where neither is efficient.

Table 1
Data and efficiencies measured from two models of an example

	Input X	Output Y	CCR efficiency	Parallel model		
				Inefficiency slack	Inefficiency score	Efficiency score
A	5	9	1	1/10	1/10	9/10
a_1	2	4		0	0	1
a_2	3	5		1/10	1/6	5/6
B	8	10	25/36	6/16	6/16	10/16
b_1	1	1		1/16	1/2	1/2
b_2	3	3		3/16	1/2	1/2
b_3	4	6		2/16	1/4	3/4

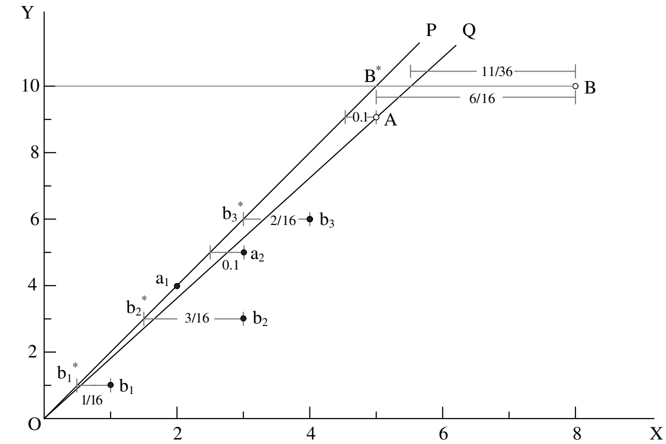

Fig. 2. The production frontier constructed from DMUs A and B with and without considering their component production units (a_1, a_2) and (b_1, b_2, b_3).

The third-to-last column of Table 1 shows the inefficiency slacks of the two DMUs and their subordinated production units calculated from Model (6). For DMU A, its inefficiency slack is 0.1, and this amount is attributed to the inefficient production unit a_2. Note that the inefficiency score of a_2 is not 0.1. According to (5), s_k^p must be divided by $\sum_{i=1}^{m}v_iX_{ik}^p$ to obtain a standardized score between 0 and 1, which is 1/6 in this case, as shown in the second-to-last column of Table 1. Complementarily, its efficiency score is 5/6. The case of DMU B is a little complicated. Its inefficiency slack is $s_B=6/16$. This inefficiency is caused by its production units b_1, b_2, and b_3 with component inefficiency slacks of 1/16, 3/16, and 2/16, respectively, as shown in Table 1 and Fig. 2. Similar to the case of DMU A, only the efficiency score of DMU B is equal to the complement of its inefficiency slack 6/16, which is 10/16. For its production units, their inefficiency slacks must be adjusted by the quantity of input $\sum_{i=1}^{m}v_iX_{ik}^p$ to obtain the inefficiency score. The resulting efficiency scores for b_1, b_2, and b_3 are 0.5, 0.5, and 0.75, respectively, as shown in the last column of Table 1. In this example, the CCR efficiency of each production unit, that is, by treating five production units as independent DMUs, is the same as their corresponding efficiency calculated from the parallel model.

Interestingly, although b_1 has the smallest inefficiency slack, it has the largest inefficiency score. This obviously is due to its small amount of input. On the other hand, b_2 has the largest amount of inefficiency slack, its inefficiency score is the same as that of b_1. One way of reducing this inefficiency slack is to reduce its input. Suppose the efficiency of b_2 is maintained at 1/2 when its input is being adjusted. The output will be proportionally reduced to 1 if its input is decreased to 1, and the inefficiency slack becomes 1/16. In this case, B will have a total input of 6, a total output of 8, and a total inefficiency slack of 4/16. Consequently, its efficiency score becomes 2/3, which is greater than its original efficiency score of 5/8 by 1/24.

Compared with the short-run and long-run concept of Kao (2000), the long-run efficiency of a production unit is obtained by minimizing the inefficiency slack of that production unit while the parallel unit efficiency of this paper is obtained by minimizing the total inefficiency slack of all production units within a DMU.

4. Forest production in Taiwan

For many organizations, their production is allocated to several independent units for efficiency reasons. Usually, geographical separation is the major consideration. Each unit in the organization applies the same inputs to produce the same outputs. Forest production is an example of this.

Taiwan is an island of 36,000 square kilometers, of which one-half are covered with forests. The forestlands are divided into eight districts, and each is further divided into four or five sub-districts called working circles (WCs). In total, there are 34 WCs. WCs are the basic unit in forest management. The forest production system is a typical parallel production system, in that each district has several subordinated WCs operating independently. Organizationally, each district is an independent unit while the working circle is not because it does not have an administrator. The Taiwan Forestry Bureau, who is in charge of the national forests of Taiwan, is interested in the efficiency of each district, rather than their subordinated working circles. The director of each district can reallocate the resources to the working circles to improve the efficiency of the district. Färe and Grosskopf (2000) build a network model to discuss this idea. However, the system efficiency calculated from their model cannot be decomposed into efficiencies of the component units.

The efficiency of forest production has been studied by Kao and Yang (1991) and Kao et al. (1993). How to reorganize forest districts to gain higher efficiency has also been investigated by Kao and Yang (1992). In this paper, we use the parallel DEA Model (6) to measure the inefficiency slacks of the eight forest districts as well as the 34 WCs. The data, as shown in Table 2, are taken from Kao (1998, 2000). There are four inputs:

(1) Land: area in thousand hectares.
(2) Labor: number of employees in persons.
(3) Expenditures: money spent each year in ten-thousand New Taiwan dollars (1000 NTD ≅ 30 USD).
(4) Initial stocks: volume of forest stock before the period of evaluation in 10,000 m^3.

The outputs considered are

(1) Timber production: timber harvested each year in cubic meters.
(2) Soil conservation: volume of forest stock in 10,000 m^3, as higher stock level leads to less soil erosion.
(3) Recreation: visitors serviced by forests every year in thousands of visits.

For each input/output the amount of a district is the sum of its subordinated WCs.

Table 2
Input and output data of Taiwan forests

Working circles	Land (1000 ha)	Inputs			Outputs		
		Labor (person)	Expenditures (10,000 NT)	Initial stocks (10,000 m^3)	Timber (m^3)	Soil cons. (10,000 m^3)	Recreation (1000 vis)
Lotung District	*175.73*	*248.33*	*1581.60*	*1604.38*	*746.04*	*1604.01*	*207.59*
1. Taipei	18.23	45.33	608.32	125.46	19.59	125.46	0.00
2. Tai-ping-shan	55.49	98.00	336.33	584.85	17.70	584.85	207.59
3. Chao-chi	31.44	51.00	263.99	147.76	0.00	147.39	0.00
4. Nan-au	28.94	27.33	166.78	263.02	38.00	263.02	0.00
5. Ho-ping	41.63	26.67	206.18	483.29	670.75	483.29	0.00
Hsinchu District	*162.81*	*316.67*	*850.05*	*2609.79*	*16823.42*	*2603.99*	*308.97*
6. Guay-shan	41.48	86.33	158.49	386.03	26.37	386.03	114.16
7. Ta-chi	29.72	58.00	260.02	638.87	42.53	638.87	181.01
8. Chu-tung	59.28	77.67	220.97	1218.07	1350.65	1214.48	13.80
9. Ta-hu	32.33	94.67	210.57	366.82	15403.87	364.61	0.00
Tungshi District	*138.42*	*310.34*	*864.42*	*2348.03*	*4778.32*	*2819.48*	*264.92*
10. Shan-chi	10.40	50.67	218.55	103.86	2842.34	165.63	0.00
11. An-ma-shan	33.64	111.33	153.07	731.43	0.00	728.19	38.98
12. Li-yang	38.01	97.67	272.32	421.41	1935.98	558.17	111.26
13. Li-shan	56.37	50.67	220.48	1091.33	0.00	1367.49	114.68
Nantou District	*211.82*	*287.32*	*1835.20*	*2352.10*	*11429.54*	*2343.86*	*0.00*
14. Tai-chung	10.57	64.33	319.51	39.12	3330.16	39.12	0.00
15. Tan-ta	52.69	49.00	340.54	688.60	1242.50	688.60	0.00
16. Pu-li	77.22	68.33	652.53	966.44	4134.43	966.44	0.00
17. Shui-li	54.29	59.33	348.33	602.24	2574.87	602.24	0.00
18. Chu-shan	17.05	46.33	174.29	55.70	147.58	47.46	0.00
Chiayi District	*139.65*	*203.00*	*215.77*	*1316.48*	*1086.00*	*1330.10*	*845.05*
19. A-li-shan	42.81	69.33	62.51	527.44	0.00	527.40	845.05
20. Fan-chi-hu	19.28	35.33	54.71	96.00	1086.00	95.97	0.00
21. Ta-pu	32.86	44.67	60.41	196.30	0.00	195.85	0.00
22. Tai-nan	44.70	53.67	38.14	496.74	0.00	510.88	0.00
Pingtung District	*196.06*	*250.33*	*1230.56*	*1588.02*	*7236.45*	*1588.02*	*939.69*
23. Chih-shan	35.64	61.33	37.92	150.90	1405.76	150.90	0.00
24. Chao-chou	70.19	62.00	188.12	624.80	1802.85	624.80	0.00
25. Liu-guay	70.96	55.67	461.42	722.46	4027.84	722.46	8.08
26. Heng-chun	19.27	71.33	543.10	89.86	0.00	89.86	931.61
Taitung District	*226.54*	*141.67*	*755.20*	*2679.98*	*8086.47*	*2679.98*	*161.38*
27. Kuan-shan	113.42	54.67	272.35	1607.90	7669.57	1607.90	57.87
28. Chi-ben	44.54	41.00	184.65	552.13	416.90	552.13	103.51
29. Ta-wu	44.03	20.33	100.70	394.03	0.00	394.03	0.00
30. Chan-kong	24.55	25.67	197.50	125.92	0.00	125.92	0.00
Hualien District	*320.43*	*284.00*	*1092.92*	*401.21*	*2263.01*	*4410.58*	*53.19*
31. Shin-chan	85.95	64.00	314.71	1074.86	17.77	1085.88	0.00
32. Nan-hua	51.60	76.00	228.40	886.07	110.28	882.20	16.50
33. Wan-yong	59.53	74.00	282.01	829.11	339.91	819.16	0.00
34. Yu-li	123.35	70.00	267.80	1611.17	1795.05	1623.34	36.69

Table 3 shows the results of this efficiency measurement, where the second column is the inefficiency slack solved from Model (6), the third column is the inefficiency score, $s_k^p/\sum_{i=1}^m v_i X_{ik}^p$, and the fourth column is the efficiency score, $1 - s_k^p/\sum_{i=1}^m v_i X_{ik}^p$. As is manifested in model derivation, the inefficiency slack of a district is the sum of those of its WCs, although there are negligible differences due to rounding errors. Of the eight districts, none is efficient. The one with the smallest inefficiency slack is Tungshi, all of its slack of 0.0629 is attributed to An-ma-shan WC, and this is the only inefficient WC out of its four WCs. The second best district is Chiayi, with an inefficiency slack of 0.0993. Of its four WCs, two are efficient and two are inefficient. There are three districts, Taitung, Hsinchu, and Pintung, where each has only one efficient WC. Their inefficiency slacks are 0.1398, 0.1771, and 0.2009, respectively. The remaining three districts, Hualien, Nantou, and Lotung, do not have efficient WCs. Their inefficiency slacks are 0.2055, 0.2266, and 0.2480, respectively. The reason why all working circles of a district can be inefficient is because the working circles of a district are compared with working circles of all other districts, rather than compared only with themselves, in calculating their efficiency scores. The rankings of the eight districts are, in sequence, Tungshi, Chiayi, Taitung, Hsinchu, Pintung, Hualien, Nantou, and Lotung.

The inefficiency slack of the WC is an absolute value expressed in the amount of inefficiency slack for the district that it belongs to. The WC with the largest inefficiency slack is Liu-guay (No. 25), with a slack of 0.1003. However, its inefficiency score after being adjusted by its aggregated input, 0.2311, is not the largest. On the contrary, Chu-shan (No. 18) has a relatively small slack of 0.0289, yet its inefficiency score after being adjusted by its aggregated input, which is 0.6539, is the largest.

The second-to-last column of Table 3 shows the CCR efficiencies of the eight districts. This measure ignores the requirement that each production unit should have an aggregated output which is smaller than the aggregated input. Ignoring this set of constraints leads to a high efficiency measure for every district. As a matter of fact, only two districts, i.e., Lotung and Nantou, are inefficient, and the remaining six are efficient. Since these two districts also have the smallest efficiency scores under the parallel model, the results from the two models are consistent. However, the parallel DEA model is more discriminative than the conventional DEA model in performance evaluation.

The 34 WCs can also be treated as independent DMUs to calculate their efficiency by applying the conventional CCR Model (1). The results are shown in the last column of Table 3. They are very

Table 3
Inefficiency slacks and two efficiency measures of Taiwan forests

Working circles	Parallel model			CCR district efficiency	CCR WC efficiency
	Inefficiency slack	Inefficiency score	Efficiency score		
Lotung District	*0.2480*	*0.2480*	*0.7520*	*0.8429*	
1. Taipei	0.0291	0.3330	0.6670		0.6681
2. Tai-ping-shan	0.0881	0.2399	0.7601		0.7803
3. Chao-chi	0.0336	0.3292	0.6708		0.6720
4. Nan-au	0.0372	0.2336	0.7664		0.7707
5. Ho-ping	0.0601	0.2114	0.7886		0.8048
Hsinchu District	*0.1771*	*0.1771*	*0.8229*	*1*	
6. Guay-shan	0.0331	0.2186	0.7814		0.7946
7. Ta-chi	0.0536	0.2151	0.7849		0.9600
8. Chu-tung	0.0905	0.2008	0.7992		0.9209
9. Ta-hu	0	0	1		1
Tungshi District	*0.0629*	*0.0629*	*0.9371*	*1*	
10. Shan-chi	0	0	1		1
11. An-ma-shan	0.0629	0.2088	0.7912		0.8923
12. Li-yang	0	0	1		1
13. Li-shan	0	0	1		1
Nantou District	*0.2266*	*0.2266*	*0.7734*	*0.9461*	
14. Tai-chung	0.0204	0.4299	0.5701		1
15. Tan-ta	0.0569	0.2072	0.7928		0.7928
16. Pu-li	0.0688	0.1787	0.8213		0.8265
17. Shui-li	0.0516	0.2074	0.7926		0.7926
18. Chu-shan	0.0289	0.6539	0.3461		0.5343
Chiayi District	*0.0993*	*0.0993*	*0.9007*	*1*	
19. A-li-shan	0	0	1		1
20. Fan-chi-hu	0.0419	0.3523	0.6477		0.8054
21. Ta-pu	0.0574	0.3145	0.6855		0.7843
22. Tai-nan	0	0	1		1
Pingtung District	*0.2009*	*0.2009*	*0.7991*	*1*	
23. Chih-shan	0.0228	0.2399	0.7601		0.8987
24. Chao-chou	0.0779	0.2176	0.7824		0.7991
25. Liu-guay	0.1003	0.2311	0.7689		0.8375
26. Heng-chun	0	0	1		1
Taitung District	*0.1398*	*0.1398*	*0.8602*	*1*	
27. Kuan-shan	0	0	1		1
28. Chi-ben	0.0631	0.2779	0.7221		0.7939
29. Ta-wu	0.0324	0.2216	0.7784		0.7955
30. Chan-kong	0.0443	0.5492	0.4508		0.7273
Hualien District	*0.2055*	*0.2055*	*0.7945*	*1*	
31. Shin-chan	0.0502	0.2041	0.7959		0.7998
32. Nan-hua	0.0427	0.2117	0.7883		0.7883
33. Wan-yong	0.0433	0.2268	0.7732		0.7732
34. Yu-li	0.0694	0.1918	0.8082		0.9524

consistent with those calculated from the parallel model, only the CCR efficiencies are greater than or equal to those calculated from the parallel model for all WCs. This is because in calculating the inefficiency slack of a WC, all other WCs in the same district must be taken into consideration in the parallel model, while in the conventional CCR model only the inefficiency slack of the WC in concern needs to be considered. There are nine WCs whose CCR efficiencies are greater than their parallel model efficiencies by more than 0.1. The largest difference occurs at Tai-chung WC (No. 14), which is 1 versus 0.5701. The reason is because sacrificing the efficiency score of Tai-chung WC can accomplish a higher efficiency score for Nantou District, the district it belongs to. If the inefficiency slack of Tai-chung is forced to zero in the parallel model, then the efficiency of Nantou District becomes 0.5577, smaller than its original efficiency score of 0.7734 by 0.2157. In this case, the efficiency score of Tai-chung WC is sacrificed by 0.4299 (=1 − 0.5701).

5. Conclusion

In this paper we develop a parallel DEA model to measure the efficiency of the system which is composed of parallel production units in that the units operate independently and the total of their respective inputs/outputs is the system's input/output. For this type of production system, the conventional DEA model treats the system as a whole while the parallel model of this paper basically treats each production unit as an independent decision making unit in measuring the relative efficiency.

By minimizing the inefficiency slack, instead of maximizing the efficiency, of a DMU, this paper is able to decompose the inefficiency slack of a DMU into the inefficiency slacks of its production units. This decomposition enables the decision maker to identify the units which are less efficient and need to make improvement. Notably, since the slacks of the component units are expressed in the absolute amount of slack of the system, only the efficiency score of the system is equal to the complement of its inefficiency slack. For the component units, their slacks must be adjusted by their aggregated input to obtain the inefficiency score. Consequently, large inefficiency slack does not necessarily imply low efficiency.

Another point to be noted is that the constraints of the parallel model are stronger than those of the conventional model; therefore, the efficiency scores calculated from the former are smaller than those calculated from the latter. As a matter of fact, only few DMUs will be efficient under the parallel model. An accompanying merit of this property is the increased discrimination power in performance evaluation. In the case of the national forests of Taiwan, the conventional model identifies two inefficient districts while the parallel model of this paper finds that all eight districts are inefficient. The case also confirms that the inefficiency slack of a district can be decomposed into the inefficiency slacks of its component WCs. By applying the idea of Kao (1994) of specifying the ranges of inputs and outputs that can be changed in finding the target point, a feasible way of improving the efficiency of a WC can be derived. Once the efficiency of the component WCs is improved, the efficiency of the district is improved accordingly.

All production units can be treated as independent DMUs to calculate their efficiency by using the conventional CCR model. This paper also finds that the CCR efficiencies are greater than or equal to those calculated from the parallel model because more slacks in addition to the slack of the production unit in concern must be considered in calculating the efficiency of that production unit. Importantly, the parallel model is able to decompose the inefficiency slack of a DMU into the sum of the inefficiency slacks of its subordinated production units. The relationship between the DMU and its production units, in mathematical terms, is specified.

The DEA model discussed in this paper is CCR; in other words, an assumption of constant returns to scale is imposed. The idea of this paper is also applicable to variable returns to scale under the BCC model (Banker et al., 1984), where the constraint of $\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0$ in Model (1) is replaced by $\sum_{r=1}^{s} u_r Y_{rj} - v_0 - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0$. By setting $v_0' = v_0/q$ and following the derivation in (3) and (4), Eq. (5) becomes $\sum_{r=1}^{s} u_r Y_{rk}^p - v_0' - \sum_{i=1}^{m} v_i X_{ik}^p + s_k^p = 0$, and this is the constraint to be used in Model (6).

In the real world a system is usually more complicated than the parallel system discussed in this paper. Castelli et al. (2004) and Färe and Grosskopf (2000) propose models to calculate the efficiency of each component of a hierarchical and network system, respectively. Insufficiently, the components are treated as independent units, without taking the relationship among the components into account. Since parallel and series are the two basic structures of a system, a complicated system could be represented by an equivalent parallel system of series components or series system of parallel components (Barlow and Proschan, 1975). This paper measures the efficiency of a parallel system and the model of Kao and Hwang (2008) measures the efficiency of a series system. Based on the ideas of these two papers, a direction for future study is to develop a network DEA model to calculate the efficiency of the whole system taking into account the relationship among its components.

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