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General decrementing service M/G/1 queue with multiple adaptive vacations

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ABSTRACT

In this paper, we introduce the multiple adaptive vacation policy and the general decrementing service rule based on the classical M/G/1 queueing systems, and obtain the P.G.F. (Probability Generating Function) of stationary queue length by using the embedded Markov chain method and regeneration cycle approach. Then, the LST (Laplace Stieltjes Transform) of stationary waiting time is also derived according to the independence between the waiting time and arrival process. At last some special cases are given to show the general properties of the new model, and some numerical results are shown to compare the mean queue length and waiting time of special cases.

1. Introduction

Many researchers have developed multiple vacations and single vacation queueing systems, and obtained many performance measures in those systems. For example, Cohen [1] studied many single server queues, and derived the P.G.F. of the queue length and LST of waiting time, and analyzed the busy period of the system. Gross etc. [2] also studied some queueing models, and analyzed performance measures. Tian [3] introduced a multiple adaptive vacation policy, and studied a multiple adaptive vacation M/G/1 model with an exhaustive service rule, then the queueing models with multiple vacations and single vacation were extended. Zhang and Tian [4] studied the discrete time queue model with multiple adaptive vacations, and obtained the P.G.F. of the queue length and waiting time. Takagi [5] studied the general decrementing service M/G/1 queue with multiple vacations, and obtained the P.G.F. of the stationary queue length and LST of the stationary waiting time, and showed the stochastic decomposition results of the above indices. As the special case, Takagi [6] studied the mean customer waiting time in a symmetric polling system, which is a pure decrementing service queueing system with multiple adaptive vacations. Levy and Yechiali [7] studied the exhaustive service M/G/1 queue, which is the special case of the general decrementing service M/G/1 queue with multiple vacations.

Few researchers pay attention to the nonexhaustive service queueing system with the multiple adaptive vacation policy by far now. The M/G/1 and Geom/G/1 gate service systems with multiple adaptive vacations and Geom/G/1 gate service system with multiple adaptive vacation are studied in [8,9], which obtained the P.G.F. of the stationary queue length and LST of waiting time, and analyzed the service period. In this paper, we study an M/G/1 queueing model with the general decrementing service and the multiple adaptive vacations, and obtained the P.G.F. of the stationary queue length and LST of waiting time. Our study shows that the general decrementing service queueing system with multiple vacations in [5,10] are special cases of our model presented in this paper. Furthermore, we compare the system performances for pure decrementing service queue with multiple vacations, the exhaustive service queue with multiple vacations and single vacation.

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The paper is organized as follows. In Section 2, we give the description of new model and its system parameters in detail. In Section 3, we embed a Markov chain, and derive the customer number of the system at the completion instant of the vacation. We further deduce the P.G.F. of the stationary queue length and LST of the stationary waiting time, and show the stochastic decomposition results of stationary measures. In Section 4, we present some numerical results. Concluding remarks are given in Section 5.

2. Model description

In the classical M/G/1 queueing system, we introduced the general decrementing service policy [5] and the multiple adaptive vacations rule [3]. Firstly, we define the general decrementing service policy. Once the service period starts, the server will keep on working until the number of customers in the system is M less than the number of customers at the start instant of the service period, or until there is no customer in the system for the service. After the service period completion instant, the server will take H vacations consecutively according to the assistant workload completed at present, and H is a positive random variable with the probability distribution h_j and the P.G.F. $H(z)$ as follows:

$$P(H=j)=h_j, j\geqslant 1,\quad H(z)=\sum_{j=1}^{\infty}h_jz^j.$$

Each consecutive vacation time $V_k(k=1,2,\ldots,H)$ is an independently and identically distributed (i.i.d.) random variable. There are two cases as follows:

(1) For a natural number $k(1\leqslant k\leqslant H)$, if there are customers arrived during the kth vacation, the vacation period will stop in advance at the kth vacation completion instant, then the system enters a new service period. When the server finishes serving customers, the system enters the next vacation according to the general decrementing service policy.

(2) If there are customers in the system at the Hth vacation completion instant, the server immediately serves some customers according to the general decrementing service policy, then enters the vacation period. If there is no customer in the system at the Hth vacation completion instant, the system enters an idle period, and waits for a new customer to arrive. If a customer arrives during the idle period, the server will enter a service period immediately, and serves some customers according to the general decrementing service. The system will continually repeat the above processes.

The basic assumptions of the new model are given as follows:

(1) Customers follow a Poisson arrival process with rate $\lambda(>0)$, which means that any neighbor interarrival time series $\{\tau_i, i\geqslant 1\}$ are independently and identically negative exponential distribution $F(t)=1-e^{-\lambda t}, t\geqslant 0$.

(2) Each customer's requiring service time series $\{B_i, i\geqslant 1\}$ follow an independent and identically general distribution $B(t), t\geqslant 0$. The mean of service time, the second moment and LST are respectively denoted as follows:

$$0<1/\mu=\int_0^{\infty}t\mathrm{d}B(t),\quad b^{(2)}=\int_0^{\infty}t^2\mathrm{d}B(t),\quad B^*(s)=\int_0^{\infty}e^{-st}\mathrm{d}B(t).$$

(3) Vacation time V is a nonnegative i.i.d. random variable with the general distribution $V(x)$, the first moment $E(V)$, the second moment $E(V^2)$ and the LST $v^*(s)$. Suppose that there is a single server in this system, and its buffer capacity is infinite. The interarrival time, the service time and the vacation time are mutually independent. The service order is First Come First served (FCFS). The model is denoted by M/G/1 (GD, MAV), where GD and MAV represent the General Decrementing service and the Multiple Adaptive Vacations respectively.

3. Analysis of system performance measures

For the general decrementing service M/G/1 queue, it will be studied by the embedded Markov chain method and the regeneration cycle approach, and the regeneration cycle approach is shown as the following lemma. Let $L_v^{(t)}$ be queue length process in M/G/1 (GD, MAV) queue.

Lemma 1. *If $L_v^{(t)}$ is a positive recurrent process, the P.G.F. of the stationary queue length L_v is given by*

$$L_v(z)=\frac{E\left(\sum_{n=1}^{\Phi}z^{L_n}\right)}{E(\Phi)} \tag{1}$$

where L_n is the number of customers at the departure instant of the nth customer in one service period, and Φ is the number of customers served in one service period.

The proof of Lemma 1 can be found in [5,10].

3.1. Number of customers in the system at the vacation completion instant

Let $Q_b^{(n)}$ be the number of customers in the system at the completion instant of the nth vacation, then $\{Q_b^{(n)}, n \geqslant 1\}$ is a Markov chain, and its transition probabilities are given as follows:

$$P_{jk} = \begin{cases} v_{k-j+M}, & k \geqslant j-M > 0 \\ (1-H(v^*(\lambda)))v_k, & j \leqslant M, k \neq 1 \\ (1-H(v^*(\lambda)))v_k + H(v^*(\lambda)), & j \leqslant M, k = 1 \\ 0, & j > M, k < j-M \end{cases} \tag{2}$$

where

$$v_j = \int_0^\infty \frac{(\lambda x)^j}{j!} e^{-\lambda x} dV(x), \quad j \geqslant 0.$$

At the same time, let $\{q_k, k \geqslant 0\}$ be the steady state distribution of Markov chain $\{Q_b^{(n)}, n \geqslant 1\}$, i.e.

$$q_k = \lim_{n\to\infty} P(Q_b^{(n)} = k), \quad k \geqslant 0.$$

The stationary probabilities satisfy the equilibrium equations as follows:

$$q_0 = v_0(1-H(v^*(\lambda)))\sum_{j=0}^{M} q_j, \tag{3}$$

$$q_1 = ((1-H(v^*(\lambda)))v_0 + H(v^*(\lambda)))\sum_{j=0}^{M} q_j + v_0 q_{M+1}, \tag{4}$$

$$q_k = (1-H(v^*(\lambda)))v_k \sum_{j=0}^{M} q_j + \sum_{j=M+1}^{k+M} v_{k-j+M} q_j, \quad k \geqslant 2. \tag{5}$$

Define the P.G.F. and the partial P.G.F. of $\{q_k, k \geqslant 0\}$

$$Q_b(z) = \sum_{k=0}^{\infty} q_k z^k, \quad Q_M(z) = \sum_{k=0}^{M} q_k z^k.$$

Multiplying Eqs. (3)–(5) by z^0, z, z^k respectively, we can derive the P.G.F. $Q_b(z)$ of $\{q_k, k \geqslant 0\}$ as follows:

$$\begin{aligned} Q_b(z) &= (1-H(v^*(\lambda)))\sum_{j=0}^{M} q_j \sum_{k=0}^{\infty} v_k z^k + H(v^*(\lambda))z\sum_{j=0}^{M} q_j + v_0 z q_{M+1} + \sum_{k=2}^{\infty} z^k \sum_{j=M+1}^{k+M} v_{k-j+M} q_j \\ &= (1-H(v^*(\lambda)))v^*(\lambda(1-z))Q_M(1) + H(v^*(\lambda))zQ_M(1) + \sum_{k=1}^{\infty} z^k \sum_{j=M+1}^{k+M} v_{k-j+M} q_j \\ &= ((1-H(v^*(\lambda)))v^*(\lambda(1-z)) + H(v^*(\lambda))z)Q_M(1) + \frac{1}{z^M} v^*(\lambda(1-z))(Q_b(z) - Q_M(z)). \end{aligned} \tag{6}$$

Solving Eq. (6), we have

$$Q_b(z) = \frac{1}{v^*(\lambda(1-z)) - z^M} \times (v^*(\lambda(1-z))Q_M(z) - ((1-H(v^*(\lambda)))v^*(\lambda(1-z)) + H(v^*(\lambda))z)z^M Q_M(1)). \tag{7}$$

We can obtain the values of $q_0, q_1, \ldots, q_M$ by applying Rouche theorem and Lagrange theorem [11,12], so that we can give the stochastic decomposition of the stationary performance measures in the steady state system.

3.2. Partial probability generation function $Q_M(z)$

If $\lambda E(V) < M$, the denominator of the right hand side of Eq. (7) has $M-1$ zeros $z_1, \ldots, z_{M-1}$, inside $|z| = 1$, and these zeros are given by Lagrange theorem as

$$z_m = \sum_{n=1}^{\infty} \frac{e^{2\pi imn}}{n!} \frac{d^{n-1}}{dz^{n-1}} v^*(\lambda(1-z))|_{z=0}, \quad m = 1, 2, \ldots, M-1.$$

Because $Q_b(z)$ is analytic onside $|z| < 1$, the numerator of the right hand side of Eq. (7) must also be zero at $z_1, \ldots, z_{M-1}$, i.e. $q_0, q_1, \ldots, q_M$ satisfy the following set of equations

$$\sum_{k=0}^{M}\left(v^*(\lambda(1-z_m))z_m^M-((1-H(v^*(\lambda)))v^*(\lambda(1-z_m))+H(v^*(\lambda))z_m)z_m^M\right)q_k=0,\quad m=1,\ldots,M-1. \tag{8}$$

Using the normalization condition $Q_b(1)=1$, we have

$$Q_M'(1)=\sum_{k=1}^{M}kq_k=\lambda E(V)-M+(M+H(v^*(\lambda))(1-\lambda E(V)))Q_M(1). \tag{9}$$

Combining Eqs. (3), (8) and (9), these $M+1$ coefficients of $Q_M(z)$ are determined by a set of $M+1$ linear equations, so $Q_b(z)$ is the known function.

3.3. Stochastic decompositions of the stationary queue length and waiting time

Theorem 1. *If $\rho<1$ and $\lambda E(V)<M$, the stationary queue length L_v in M/G/1 (GD, MAV) queue can be decomposed into three independent random variables:*

$$L_v=L+L_d+L_r,$$

where L is the stationary queue length in the classical M/G/1 queue, and the P.G.F. of L can be seen in [5], and the P.G.F. of the additional queue length L_d and L_r are given by

$$L_d(z)=\frac{1-H(v^*(\lambda))z-\frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}(v^*(\lambda(1-z))-v^*(\lambda))}{\left(H(v^*(\lambda))+\frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}\lambda E(V)\right)(1-z)},$$

$$L_r(z)=\Bigg(\frac{\beta}{(v^*(\lambda(1-z))-z^M)(1-v^*(\lambda(1-z))+H(v^*(\lambda))(v^*(\lambda(1-z))-(1-v^*(\lambda))z-v^*(\lambda)))}\times(Q_M(z)(1-v^*(\lambda(1-z)))-Q_M(1)((1-v^*(\lambda(1-z)))z^M+H(v^*(\lambda))(z-v^*(\lambda(1-z)))(1-z^M)))\Bigg). \tag{10}$$

Proof. Let b be the mean number of customers served during a busy period in the classical M/G/1 queue, then we have

$$b=\frac{1}{1-\rho}, \tag{11}$$

where the proof of Eq. (11) can be found in [5]. And let Φ be the number of customers served in a service period of the M/G/1 (GD, MAV). If $Q_b=k, 1\leqslant k\leqslant M$, the length of the service period is k times of the length of a busy period in the classical M/G/1 queue; If $Q_b=k\geqslant M$, the length of service period is M times of the length of a busy period in the classical M/G/1 queue. We have

$$\begin{aligned}E(\Phi)&=b\sum_{k=1}^{M}kq_k+Mb\sum_{k=M+1}^{\infty}q_k=\frac{1}{1-\rho}Q_M'(1)+\frac{M(1-Q_M(1))}{1-\rho}\\&=\frac{1}{1-\rho}(\lambda E(V)-M+M(1-Q_M(1))+(M+H(v^*(\lambda))(1-\lambda E(V)))Q_M(1))\\&=\frac{1}{1-\rho}(\lambda E(V)+H(v^*(\lambda))(1-\lambda E(V))Q_M(1)).\end{aligned} \tag{12}$$

In order to derive the P.G.F. of the stationary queue length by applying the regeneration cycle approach, we discuss $E(\sum_{n=1}^{\Phi}z^{L_n})$, where L_n is the number of customers in the system at the departure instant of the nth customer in a service period, and we consider Last Come First Served (LCFS) discipline. If the busy period starts with j customers in the classical M/G/1 queue, $j\geqslant 1$, then $E(\sum_{n=1}^{\Phi}z^{L_n})$ is given by

$$E\left(\sum_{n=1}^{\Phi}z^{L_n}\right)=z^{j-1}\frac{(1-z)B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z},\quad j\geqslant 1. \tag{13}$$

If $k\leqslant M$, the service period starting with k customers can be decomposed into k standard M/G/1 busy periods starting with kth, $(k-1)$th, …, 1th customer, respectively. Combining Eq. (13), we have

$$q_k\left(\sum_{j=1}^{k}z^{j-1}\right)\frac{(1-z)B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z}=\frac{q_k(1-z^k)B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z}. \tag{14}$$

If $k>M$, the service period is decomposed into M standard M/G/1 busy periods starting with kth, $(k-1)$th, …, $(k-M+1)$th customer respectively. Combining Eq. (13), we have

$$q_k\left(\sum_{j=k-M+1}^{k} z^{j-1}\right)\frac{(1-z)B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z}=\frac{q_k(z^{k-M}-z^k)B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z}. \tag{15}$$

According to Total Probability theorem, we have

$$\begin{aligned}E\left(\sum_{n=1}^{\Phi} z^{L_n}\right)&=\left(\sum_{k=1}^{M} q_k(1-z^k)+\sum_{k=M+1}^{\infty} q_k(z^{k-M}-z^k)\right)\frac{B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z}\\&=\frac{Q_M(1)-Q_M(z)+(z^{-M}-1)(Q_b(z)-Q_M(z))}{B^*(\lambda(1-z))-z}B^*(\lambda(1-z))\\&=\frac{Q_M(1)-Q_b(z)+z^{-M}(Q_b(z)-Q_M(z))}{B^*(\lambda(1-z))-z}B^*(\lambda(1-z)).\end{aligned} \tag{16}$$

Substituting Eq. (7) into Eq. (16), we obtain

$$\begin{aligned}E\left(\sum_{n=1}^{\Phi} z^{L_n}\right)&=\frac{B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z}\frac{1}{v^*(\lambda(1-z))-z^M}\times(Q_M(z)(1-v^*(\lambda(1-z)))-Q_M(1)((1-v^*(\lambda(1-z)))z^M\\&\quad+H(v^*(\lambda))(z-v^*(\lambda(1-z)))(1-z^M))).\end{aligned} \tag{17}$$

Using the regeneration cycle approach, see Lemma 1, we have

$$\begin{aligned}L_v(z)&=\frac{E\left(\sum_{n=1}^{\Phi} z^{L_n}\right)}{E(\Phi)}\\&=\frac{(1-\rho)(1-z)B^*(\lambda(1-z))}{B^*(\lambda(1-z))-z}\times\frac{1-H(v^*(\lambda))z-\frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}(v^*(\lambda(1-z))-v^*(\lambda))}{\left(H(v^*(\lambda))+\frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}\lambda E(V)\right)(1-z)}\\&\quad\times\left(\frac{\beta}{(v^*(\lambda(1-z))-z^M)(1-v^*(\lambda(1-z))+H(v^*(\lambda))(v^*(\lambda(1-z))-(1-v^*(\lambda))z-v^*(\lambda)))}\right.\\&\quad\left.\times(Q_M(z)(1-v^*(\lambda(1-z)))-Q_M(1)((1-v^*(\lambda(1-z)))z^M+H(v^*(\lambda))(z-v^*(\lambda(1-z)))(1-z^M)))\right)=L(z)L_d(z)L_r(z).\end{aligned} \tag{18}$$

where

$$\beta=\frac{\lambda E(V)+H(v^*(\lambda))(1-v^*(\lambda)-\lambda E(V))}{\lambda E(V)+H(v^*(\lambda))(1-\lambda E(V))Q_M(1)},$$

therefore we obtain the P.G.F. of the stationary queue length in the system. □

The proof of Theorem 1 indicates that the additional queue length can be decomposed into two parts in M/G/1 (GD, MAV) queue, where L_d is the additional queue length of M/G/1 (E, MAV), and L_r is the additional queue length which is caused by the general decrementing service. Using L'Hospital rule, we easily obtain the mean $E(L_v)$.

According to Theorem 1, and using the relationship between the queue length and the waiting time, we can obtain the stochastic decomposition property for the stationary waiting time.

Theorem 2. *If $\rho < 1$ and $\lambda E(V) < M$, the stationary waiting time W_v can be decomposed into three independent random variables in M/G/1 (GD, MAV) queue,*

$$W_v = W + W_d + W_r,$$

where W is the stationary waiting time in the classical M/G/1 queue, the LST of W can be seen in [5], and the LST of additional delay W_d and W_r are given by

$$\begin{aligned}W_d^*(s)&=\frac{\lambda-H(v^*(\lambda))(\lambda-s)-\lambda\frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}(v^*(s)-v^*(\lambda))}{\left(H(v^*(\lambda))+\frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}\lambda E(V)\right)s},\\W_r^*(s)&=\left(\frac{\frac{\beta}{\lambda^M v^*(s)-(\lambda-s)^M}}{\lambda(1-v^*(s))+H(v^*(\lambda))(\lambda(v^*(s)-v^*(\lambda))-(1-v^*(\lambda))(\lambda-s))}\right.\\&\quad\left.\times\left(\lambda^M Q_M\left(\frac{\lambda-s}{\lambda}\right)(1-v^*(s))-Q_M(1)(\lambda(1-v^*(s))(\lambda-s)^M+H(v^*(\lambda))(\lambda(1-v^*(s))-s)(\lambda^M-(\lambda-s)^M))\right)\right).\end{aligned} \tag{19}$$

Proof. In M/G/1 (GD, MAV) queue, the waiting time of a customer is independent of the arrival process of this customer after its arrival instant in FCFS order. So the stationary queue length at the completion instant of a service is composed of the number of customers arrived during its waiting time W_v and its service time S, we have

$$L_v(z) = W_v^*(\lambda(1-z))B^*(\lambda(1-z)). \tag{20}$$

The proof of Eq. (20) can be found in [5], and substituting $L_v(z)$ of Theorem 1 into Eq. (20), and letting $s = \lambda(1-z)$, we obtain

$$W_v^*(s) = \frac{(1-\rho)s}{s-\lambda(1-B^*(s))} \tag{21}$$
$$\times \frac{\lambda - H(v^*(\lambda))(\lambda - s) - \lambda \frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}(v^*(s)-v^*(\lambda))}{\left(H(v^*(\lambda)) + \frac{1-H(v^*(\lambda))}{1-v^*(\lambda)}\lambda E(V)\right)s}$$
$$\times \left(\frac{\frac{\beta}{\lambda^M v^*(s)-(\lambda-s)^M}}{\lambda(1-v^*(s)) + H(v^*(\lambda))(\lambda(v^*(s)-v^*(\lambda)) - (1-v^*(\lambda))(\lambda-s))} \right.$$
$$\times \left(\lambda^M Q_M\left(\frac{\lambda-s}{\lambda}\right)(1-v^*(s)) - Q_M(1)(\lambda(1-v^*(s))(\lambda-s)^M \right.$$
$$\left.\left. + H(v^*(\lambda))(\lambda(1-v^*(s))-s)(\lambda^M-(\lambda-s)^M)\right)\right)$$
$$= W^*(s)W_d^*(s)W_r^*(s). \tag{22}$$

Therefore, Eq. (19) is the LST of the additional delay. □

Similarly, deriving Eq. (21) and using L'Hospital rule, we easily obtain the mean $E(W_v)$ of the stationary waiting time in M/G/1 (GD, MAV) queue.

Specially, when the positive random variable H follows the different distribution, we can obtain some concrete M/G/1 queue with vacations. For example, if $H = \infty$, this queueing system corresponds to the general decrementing service M/G/1 queue with multiple vacations – M/G/1 (GD, MV); if $H = 1$, it corresponds to the general decrementing service M/G/1 queue with single vacation – M/G/1 (GD, SV); if H follows another distributions (geometric distribution, Possion distribution, etc.), it corresponds to another models. If $M = 1$, this queueing system corresponds to M/G/1 (PD, MAV), if $M = \infty$, it corresponds to M/G/1 (E, MAV), etc. The performance measures of these models all can be obtained by substituting derived $H(v^*(\lambda))$, $Q_M(z)$ and supposed M into Theorems 1 and 2.

4. Numerical results

In this section, we present some numerical results that provide insight into the system behavior. Using the equations presented in Section 3, we can numerically compare the performance measures of the systems for three different M/G/1 (GD, MAV) queueing models: the pure decrementing service M/G/1 queue with multiple vacations, the exhaustive service M/G/1 queue with multiple vacations and single vacation. Here we assume that the service time S and the time length V of a vacation follow exponential distributions, i.e., S follows an exponential distribution with parameter $\mu = 0.8$. V follows an exponential distribution with parameter $\theta = 0.6$. As we presented in Section 3, if $H \to \infty$ and $M = 1$, the model corresponds to an M/G/1 (PD, MV) Queue. If $H \to \infty$ and $M \to \infty$, the model corresponds to an M/G/1 (E, MV) queue. If $H = 1$ and $M \to \infty$, the model corresponds to an M/G/1 (E, SV) queue. By using Eqs. (10) and (19), we can derive the mean queue length and waiting time. Suppose that traffic intensity ρ ranges from 0.15 to 0.6.

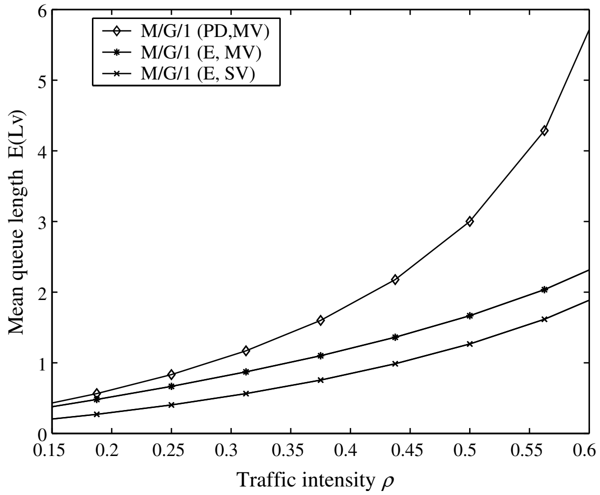

Fig. 1. Mean queue length $E(L_v)$ versus traffic intensity ρ.

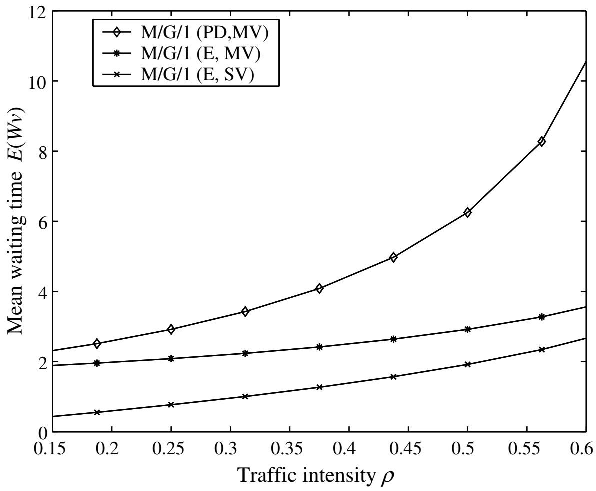

Fig. 2. Mean waiting time $E(W_v)$ versus traffic intensity ρ.

Fig. 1 shows the mean queue length $E(L_v)$ as a function of the traffic intensity ρ with three cases of H, i.e., M/G/1 (PD, MV) queue, M/G/1 (E, MV) queue and M/G/1 (E, SV) queue. We can find that when ρ increases, $E(L_v)$ increases to a high level for all the cases. It is shown that the larger ρ is, the higher the possibility that there will be customers arriving during the server cycle. We also note that the mean queue length $E(L_v)$ of M/G/1 (PD, MV) is larger than that of M/G/1 (E, MV) and M/G/1 (E, SV). This is from the fact that the longer the vacation times are, the larger the mean queue length $E(L_v)$ will be. And because when there are customers in M/G/1 queue with the pure decrementing service policy, the server can take vacations, the mean queue length is larger than that of queueing model with the exhaustive service policy.

Fig. 2 shows how the mean waiting time $E(W_v)$ changes with the traffic intensity ρ for the three different cases of H, i.e., M/G/1 (PD, MV) queue, M/G/1 (E, MV) queue and M/G/1 (E, SV) queue. We can find that when ρ increases, $E(W_v)$ increases to a high level. We can find that the greater ρ is, the higher the possibility that there will be customers arriving during the server cycle, then the mean waiting time will be greater. We also note that the mean waiting time $E(W_v)$ of M/G/1 (PD, MV) is longer than that of M/G/1 (E, MV) and M/G/1 (E, SV). There is the similar explanation with $E(L_v)$. And because when there are customers in M/G/1 queue with the pure decrementing service policy, the server can take vacations, the mean waiting time is larger than that of queueing model with the exhaustive service policy.

5. Conclusion

We presented a detailed description on the general decrementing service queueing system with multiple adaptive vacations. We gave out the P.G.F. of the stationary queue length by using the embedded Markov chain method and the regeneration cycle approach, the LST of the waiting time according to the independence between the waiting time and the arrival process, and their stochastic decomposition results. We also obtained the P.G.F. of the additional queue length and the LST of the additional delay. At last we pointed out some special examples, which showed the universality of the queue system. In the future research we will give the practical numerical examples and use these results in computer network.

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