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Parameter-dependent H_∞ filter design for LPV systems and an autopilot application

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ABSTRACT

This paper considers the design problem of parameter dependent H_∞ filters for linear parameter varying (LPV) systems whose parameters are measurable. Conditions for existence of parameter-dependent Lyapunov function are proposed via parametrical linear matrix inequality (LMI) constraints. Based on the solutions to the LMIs, an algorithm for the gain matrices of LPV filter is presented. The design method is applied to a missile system to demonstrate the effectiveness.

1. Introduction

LPV systems are a class of linear systems whose state-space matrices depend on time-varying parameters. LPV systems are widely used for describing practical systems such as missiles [1–3], aircrafts and spacecrafts [4–6], energy production systems [7–9], inverted pendulum [10], and automated vehicles [11]. Filter analysis and design in LPV systems have attracted considerable investigation over the last decade and several methods of designing filters have been proposed [12–15]. At early stage, parameter-independent Lyapunov function is presented [16]. Later, parameter-dependent Lyapunov function method is proposed to achieve less conservatism for the LPV systems whose parameters vary in a polytopic domain using parametrically affine Lyapunov function method [12,14,17]. Extension research for LPV filter in [17,18] studies reduced-order filtering. Result in [19] presents an improved filtering method for discrete-time systems.

Parameters in LPV systems can be viewed as parametric uncertainty or parameters which can be measured in real time during system operation [20]. In [13,14], the parameters are considered as uncertainty and the parameter-dependent LMI conditions for the existence of parametrically affine and parameter-independent Lyapunov function are presented if the parameters vary in polytopic region. In [21], the parameters are assumed to be measured and an H_∞ controller is designed for the LPV system, but no filter is designed.

Unlike in [13,14] where parameters are required to be in a polytopic region, here we only need parameters to be in a compact set. In this paper, an H_∞ filter is designed for an LPV system. Conditions of existence of parameter-dependent Lyapunov function are formulated via LMI constraints and an algorithm for LPV filter gain matrices based on the solutions to the LMI conditions is presented. Then we demonstrate the effectiveness of our method in designing an LPV filter for a missile pitch-axis autopilot model which needs state estimation to design controller. For the same missile system, a non-LPV filter is designed with no stability analysis given in [2], and we provide an LPV filter design method.

The rest of this paper is organized as follows. Section 2 presents the preliminaries. Section 3 gives the main results for the LPV filter design. An application of the proposed design method to the missile pitch-axis autopilot model is given in Section 4. Section 5 contains conclusion.

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Notation: I and 0 denote the identity matrix and zeros matrix with proper dimensions. $*$ denotes the symmetric part in a matrix. $Ker(\cdot)$ is the manipulation of solving the null matrix.

2. Preliminaries

Consider the following LPV system

$$\begin{aligned} \dot{x} &= A(\rho)x + B_1(\rho)w, \\ z &= C_1(\rho)x + D_{11}(\rho)w, \\ y &= C_2(\rho)x + D_{21}(\rho)w, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state with $x = 0$ at $t = 0, y \in \mathbb{R}^{n_y}$ is the measured output, $z \in \mathbb{R}^{n_z}$ is the signal to be estimated, $w \in \mathbb{R}^{n_w}$ is the disturbance input and $w \in L_2$. $\rho = [\rho_1, \ldots, \rho_s]^T$ is assumed to lie in a compact set $\mathcal{P} \subset \mathbb{R}^s$ with its parameter variation rate bounded by $\underline{v}_k \leqslant \dot{\rho}_k \leqslant \overline{v}_k, k = 1, 2, \ldots, s$, i.e., $\dot{\rho} \in \mathcal{P}_d$.

A full order parameter-dependent filter to be designed is of the form:

$$\begin{aligned} \dot{x}_f &= A_f(\rho)x_f + B_f(\rho)y, \\ z_f &= C_f(\rho)x_f + D_f(\rho)y, \end{aligned} \tag{2}$$

where $x_f \in \mathbb{R}^n$ is the filter state with $x_f = 0$ at $t = 0$ and $z_f \in \mathbb{R}^{n_z}$ is the estimated signal of z.

Given (1) and (2), the connected system in Fig. 1 is expressed as:

$$\begin{aligned} \dot{\sigma} &= \widehat{A}(\rho)\sigma + \widehat{B}(\rho)w, \\ e &= \widehat{C}(\rho)\sigma + \widehat{D}(\rho)w, \end{aligned} \tag{3}$$

where $\sigma = \begin{bmatrix} x^T & x_f^T \end{bmatrix}^T, e = z - z_f$ and

$$\widehat{A}(\rho) = \begin{bmatrix} A(\rho) & 0 \\ B_f(\rho)C_2(\rho) & A_f(\rho) \end{bmatrix}, \quad \widehat{B}(\rho) = \begin{bmatrix} B_1(\rho) \\ B_f(\rho)D_{21}(\rho) \end{bmatrix},$$

$$\widehat{C}(\rho) = [\, C_1(\rho) - D_f(\rho)C_2(\rho) \quad -C_f(\rho) \,], \quad \widehat{D}(\rho) = D_{11}(\rho) - D_f(\rho)D_{21}(\rho).$$

The filtering problem to be dealt with is stated as follows:

Problem 1. Find $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$, $D_f(\rho)$ of the filter (2) such that the estimation error system (3) is quadratically stable when $w = 0$, and an upper bound γ to the H_∞ estimation error performance is assured, i.e.,

$$\sup_{w \in L_2, w \neq 0} \frac{\|e\|_2}{\|w\|_2} < \gamma, \quad \rho \in \mathcal{P}, \quad \dot{\rho} \in \mathcal{P}_d. \tag{4}$$

We now split Problem 1 into the following two subproblems:

Subproblem 1. Propose the conditions for the existence of parameter-dependent Lyapunov function $x^TP(\rho)x$ and H_∞ performance index γ such that system (3) is quadratically stable when $w = 0$ and (4) is satisfied.

Subproblem 2. According to the solved parameter-dependent matrix $P(\rho)$ and the index γ, find $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$, $D_f(\rho)$ of the filter (2).

The following lemmas are required when dealing with the problem above.

Lemma 1 (Bounded Real Lemma [21]). *If there exist a positive definite matrix $P(\rho)$ and a positive number γ such that*

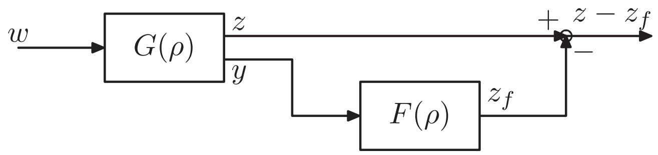

Fig. 1. Block diagram of LPV filter $F(\rho)$ for an LPV system $G(\rho)$.

$$\begin{bmatrix} \widehat{A}^T(\rho)P(\rho)+P(\rho)\widehat{A}(\rho)+\sum_{k=1}^{s}\frac{\partial P(\rho)}{\partial \rho_k}\dot{\rho}_k & P(\rho)\widehat{B}(\rho) & \widehat{C}^T(\rho) \\ * & -\gamma I & \widehat{D}^T(\rho) \\ * & * & -\gamma I \end{bmatrix} < 0 \quad (5)$$

holds for all $\rho \in \mathcal{P}, \dot{\rho} \in \mathcal{P}_d$, then system (3) is quadratically stable and satisfies (4).

Lemma 2 (Elimination Lemma [22]). *Given a symmetric matrix $\Psi \in \mathbb{R}^{m\times m}$ and two matrices G, H of column dimension m, consider the problem of finding some matrix Θ of compatible dimensions such that*

$$\Psi + G^T\Theta^T H + H^T\Theta G < 0. \quad (6)$$

Denote by N_G, N_H any matrices whose columns form bases of the null spaces of G and H, respectively. Then (6) is solvable for Θ iff

$$\begin{cases} N_G^T\Psi N_G < 0, \\ N_H^T\Psi N_H < 0. \end{cases} \quad (7)$$

Lemma 3 [23]. *Given two positive definite matrices R and S, a positive integer m, there exist matrices R_2, S_2, and symmetric matrices R_3, S_3 such that*

$$\begin{bmatrix} R & R_2 \\ R_2^T & R_3 \end{bmatrix} > 0,$$

and

$$\begin{bmatrix} R & R_2 \\ R_2^T & R_3 \end{bmatrix}^{-1} = \begin{bmatrix} S & S_2 \\ S_2^T & S_3 \end{bmatrix},$$

iff

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \geqslant 0, \quad \text{and} \quad rank\left(\begin{bmatrix} R & I \\ I & S \end{bmatrix}\right) \leqslant n+m.$$

3. Main results

In this section, the main results are presented. Theorem 1 provides sufficient conditions of existence of a parameter-dependent Lyapunov function for error dynamic system (3) via LMI formulation. Based on the solutions to the LMI conditions in Theorem 1, Algorithm 1 provides an algorithm of solving the gain matrices of LPV filter (2). Theorem 2 presents that the filter achieved from Algorithm 1 makes the system (3) satisfy the Bounded Real Lemma.

Theorem 1. *Given an LPV system (1). If there exist symmetric positive definite matrices $R(\rho) \in \mathbb{R}^{n\times n}, S(\rho) \in \mathbb{R}^{n\times n}$ such that for any $\rho \in \mathcal{P}, \dot{\rho} \in \mathcal{P}_d$, the following LMIs are satisfied:*

$$N_S^T \begin{bmatrix} \left\{ \begin{array}{l} A^T(\rho)S(\rho)+S(\rho)A(\rho) \\ +\sum_{k=1}^{s}\frac{\partial S(\rho)}{\partial \rho_k}\{\underline{v}_k, \bar{v}_k\} \end{array} \right\} & * & * \\ B_1^T(\rho)S(\rho) & -\gamma I & * \\ C_1(\rho) & D_{11}(\rho) & -\gamma I \end{bmatrix} N_S < 0, \quad (8)$$

$$\begin{bmatrix} R(\rho)A^T(\rho)+A(\rho)R(\rho)-\sum_{k=1}^{s}\frac{\partial R(\rho)}{\partial \rho_k}\{\underline{v}_k, \bar{v}_k\} & * \\ B_1^T(\rho) & -\gamma I \end{bmatrix} < 0, \quad (9)$$

$$\begin{bmatrix} R(\rho) & * \\ I & S(\rho) \end{bmatrix} \geqslant 0, \quad (10)$$

where $N_S = Ker([C_2(\rho) \quad D_{21}(\rho) \quad 0])$, then Subproblem 1 for Problem 1 is solved.

Proof. It is easy to see that LMI (5) is equivalent to

$$\begin{bmatrix} A_0^T(\rho)P(\rho)+P(\rho)A_0(\rho)+\sum_{k=1}^{s}\frac{\partial P(\rho)}{\partial\rho_k}\dot{\rho}_k & P(\rho)B_0(\rho) & C_0^T(\rho) \\ * & -\gamma I & D_{11}^T(\rho) \\ * & * & -\gamma I \end{bmatrix}$$
$$+\begin{bmatrix} P(\rho)\bar{I} \\ 0 \\ \bar{J} \end{bmatrix}\Omega_f(\rho,\dot{\rho})\begin{bmatrix}\overline{C}_2(\rho) & \overline{D}_{21}(\rho) & 0\end{bmatrix}+\begin{bmatrix}\overline{C}_2^T(\rho) \\ \overline{D}_{21}^T(\rho) \\ 0\end{bmatrix}\Omega_f^T(\rho,\dot{\rho})\begin{bmatrix}\bar{I}^TP(\rho) & 0 & \bar{J}^T\end{bmatrix}<0, \tag{11}$$

where

$$A_0(\rho)=\begin{bmatrix}A(\rho) & 0 \\ 0 & 0\end{bmatrix},\quad B_0(\rho)=\begin{bmatrix}B_1(\rho) \\ 0\end{bmatrix},\quad C_0(\rho)=\begin{bmatrix}C_1(\rho) & 0\end{bmatrix},$$

$$\overline{C}_2(\rho)=\begin{bmatrix}0 & I \\ C_2(\rho) & 0\end{bmatrix},\quad \overline{D}_{21}(\rho)=\begin{bmatrix}0 \\ D_{21}(\rho)\end{bmatrix},\quad \bar{J}=\begin{bmatrix}0 & -I\end{bmatrix},$$

$$\bar{I}=\begin{bmatrix}0 & 0 \\ I & 0\end{bmatrix},\quad \Omega_f(\rho,\dot{\rho})=\begin{bmatrix}A_f(\rho,\dot{\rho}) & B_f(\rho) \\ C_f(\rho) & D_f(\rho)\end{bmatrix}.$$

By Lemma 2, (11) is equivalently transformed into

$$N_H^T\Psi N_H<0, \tag{12}$$
$$N_G^T\Psi N_G<0, \tag{13}$$

where

$$\Psi=\begin{bmatrix} A_0^T(\rho)P(\rho)+P(\rho)A_0(\rho)+\sum_{k=1}^{s}\frac{\partial P(\rho)}{\partial\rho_k}\dot{\rho}_k & * & * \\ B_0^T(\rho)P(\rho) & -\gamma I & * \\ C_0(\rho) & D_{11}(\rho) & -\gamma I\end{bmatrix},$$

$$H=\begin{bmatrix}\overline{C}_2(\rho) & \overline{D}_{21}(\rho) & 0\end{bmatrix},\quad G=\begin{bmatrix}\bar{I}^TP(\rho) & 0 & \bar{J}^T\end{bmatrix}. \tag{14}$$

Define

$$P(\rho)=\begin{bmatrix}S(\rho) & N(\rho) \\ N^T(\rho) & \widehat{S}(\rho)\end{bmatrix},\quad P^{-1}(\rho)=\begin{bmatrix}R(\rho) & M(\rho) \\ M^T(\rho) & \widehat{R}(\rho)\end{bmatrix},$$
$$\overline{G}=\begin{bmatrix}\bar{I}^T & 0 & \bar{J}^T\end{bmatrix},\quad \overline{P}=\begin{bmatrix}P(\rho) & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I\end{bmatrix}, \tag{15}$$

then it is easy to show that (12) is equivalent to (8). Therefore, we have $G=\overline{G}\overline{P}, Ker(G)=\overline{P}, Ker(\overline{G})=\overline{P}^{-1}N_{\overline{G}}$ and $N_G^T\Psi N_G=N_{\overline{G}}^T\overline{P}^{-1}\Psi\overline{P}^{-1}N_{\overline{G}}=N_{\overline{G}}^T\Phi N_{\overline{G}}$, where

$$\Phi=\begin{bmatrix} P^{-1}(\rho)A_0^T(\rho)+A_0(\rho)P^{-1}(\rho)-\sum_{k=1}^{s}\frac{\partial P^{-1}(\rho)}{\partial\rho_k}\dot{\rho}_k & * & * \\ B_0^T(\rho) & -\gamma I & * \\ C_0(\rho)P^{-1}(\rho) & D_{11}(\rho) & -\gamma I\end{bmatrix}.$$

From the definition of $P^{-1}(\rho)$ and $Ker(\overline{G})=\begin{bmatrix}I & 0 & 0 & 0 \\ 0 & 0 & I & 0\end{bmatrix}$, (13) is equivalent to (9). According to Lemma 3, (10) guarantees the $R(\rho)$ and $S(\rho)$ can construct $P(\rho)$ as in the form of (15). The LMIs in Theorem 1 implies the condition of Lemma 1 is satisfied. □

For Subproblem 2, we propose the following algorithm to construct the filter space matrices $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$ and $D_f(\rho)$.

Algorithm 1. Given the solutions $R(\rho)$, $S(\rho)$ and γ to LMIs (8)–(10), the filter can be constructed by the following steps:

(1) Compute solution $D_f(\rho)$ such that

$$\Delta=\begin{bmatrix}\gamma I & -\widehat{D}^T(\rho) \\ * & \gamma I\end{bmatrix}>0.$$

(2) Compute solutions $\widetilde{B}_f(\rho)$ and $\widetilde{C}_f(\rho)$ to the linear matrix equations as follows:

$$\begin{bmatrix} 0 & D_{21}(\rho) & 0 \\ * & -\gamma I & \widehat{D}^T(\rho) \\ * & * & -\gamma I \end{bmatrix} \begin{bmatrix} \widetilde{B}_f^T(\rho) \\ \delta_B \end{bmatrix} = - \begin{bmatrix} C_2(\rho) \\ B_1^T(\rho)S(\rho) \\ \overline{C}(\rho) \end{bmatrix}, \tag{16}$$

$$\begin{bmatrix} 0 & 0 & I \\ * & -\gamma I & \widehat{D}^T(\rho) \\ * & * & -\gamma I \end{bmatrix} \begin{bmatrix} -\widetilde{C}_f(\rho) \\ \delta_C \end{bmatrix} = - \begin{bmatrix} 0 \\ B_1^T(\rho) \\ C_1(\rho)R(\rho) \end{bmatrix}, \tag{17}$$

where $\overline{C}(\rho) = C_1(\rho) - D_f(\rho)C_2(\rho)$. If there is singularity in (16) and (17), $\widetilde{B}_f(\rho)$ and $\widetilde{C}_f(\rho)$ can be achieved by solving the following inequalities:

$$H_{22} + L_2\Delta^{-1}L_2^T < 0, \tag{18}$$

$$H_{11} + L_1\Delta^{-1}L_1^T < 0, \tag{19}$$

where

$$H_{11} = A(\rho)R(\rho) + R(\rho)A^T(\rho) - \dot{R}(\rho),$$
$$H_{22} = A^T(\rho)S(\rho) + S(\rho)A(\rho) + \dot{S}(\rho) + \widetilde{B}_f(\rho)C_2(\rho) + C_2^T(\rho)\widetilde{B}_f^T(\rho),$$
$$L_1 = [B_1(\rho)\ R(\rho)C_1^T(\rho) - \widetilde{C}_f^T(\rho)],$$
$$L_2 = [S(\rho)B_1(\rho) + \widetilde{B}_f(\rho)D_{21}(\rho)\ C_1^T(\rho) - C_2^T(\rho)D_f^T(\rho)].$$

(3) The matrix $\widetilde{A}_f(\rho, \dot{\rho})$ is

$$\widetilde{A}_f(\rho, \dot{\rho}) = -A^T(\rho) + (S(\rho)\dot{R}(\rho) + N(\rho)\dot{M}^T(\rho)) - \left[S(\rho)B_1(\rho) + \widetilde{B}_f(\rho)D_{21}(\rho)\ \overline{C}^T(\rho)\right]\Delta^{-1}\left[B_1(\rho)\ R(\rho)C_1^T(\rho) - \widetilde{C}_f^T(\rho)\right]^T.$$

Solve for $N(\rho)$, $M(\rho)$ from the equation

$$I - R(\rho)S(\rho) = N(\rho)M^T(\rho). \tag{20}$$

Then the matrices $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$ in filter (2) are

$$C_f(\rho) = \left(\widetilde{C}_f(\rho) - D_f(\rho)C_2(\rho)R(\rho)\right)M^{-T}(\rho),$$
$$B_f(\rho) = N^{-1}(\rho)\widetilde{B}_f(\rho),$$
$$A_f(\rho, \dot{\rho}) = N^{-1}(\rho)\left[\widetilde{A}_f(\rho, \dot{\rho}) - \widetilde{B}_f(\rho)C_2(\rho)R(\rho) - S(\rho)A(\rho)R(\rho) + (S(\rho)\dot{R}(\rho) + N(\rho)\dot{M}^T(\rho))\right]M^{-T}(\rho),$$

where $\dot{R}(\rho)$ and $\dot{M}^T(\rho)$ denote $\sum_{k=1}^{s}\frac{\partial R(\rho)}{\partial \rho_k}\dot{\rho}_k$ and $\sum_{k=1}^{s}\frac{\partial M^T(\rho)}{\partial \rho_k}\dot{\rho}_k$, respectively.

The following theorem is presented to state that the filter achieved from Algorithm 1 solves Problem 1, i.e., the inequality (5) can be guaranteed.

Theorem 2. *Given the solutions $R(\rho)$, $S(\rho)$ and γ to LMIs (8)–(10). The filter achieved from Algorithm 1 guarantees the system (3) satisfy the inequality (5).*

Proof. Define $X_1(\rho)$ and $X_2(\rho)$ as

$$X_1(\rho) = \begin{bmatrix} R(\rho) & I \\ M(\rho) & 0 \end{bmatrix}, \quad X_2(\rho) = \begin{bmatrix} I & S(\rho) \\ 0 & N^T(\rho) \end{bmatrix}.$$

According to (20), we have $X_1(\rho)P(\rho) = X_2(\rho)$. For LMI (5), by premultiplying its first row and postmultiplying its first column by $X_1^T(\rho)$ and $X_1(\rho)$ respectively, it becomes

$$\begin{bmatrix} \Pi_{11}(\rho) & \Pi_{12}(\rho) \\ \Pi_{12}^T(\rho) & -\Delta \end{bmatrix} + \begin{bmatrix} X_2^T(\rho)\bar{I} \\ 0 \\ \bar{J} \end{bmatrix} \widetilde{\Omega}_f(\rho, \dot{\rho})\left[\overline{C}_2(\rho)X_1(\rho)\ \overline{D}_{21}(\rho)\ 0\right] + \begin{bmatrix} X_1^T(\rho)\overline{C}_2^T(\rho) \\ \overline{D}_{21}^T(\rho) \\ 0 \end{bmatrix} \widetilde{\Omega}_f^T(\rho, \dot{\rho})\left[\bar{I}^T X_2(\rho)\ 0\ \bar{J}^T\right] < 0, \tag{21}$$

where

$$\Pi_{11}(\rho) = \begin{bmatrix} A(\rho)R(\rho) + R(\rho)^T A(\rho) - \dot{R}(\rho) & A(\rho) + R(\rho)A^T(\rho)S(\rho) - \left(\dot{R}(\rho)S(\rho) + \dot{M}(\rho)N^T(\rho)\right) \\ A^T(\rho) + S(\rho)A(\rho)R(\rho) - \left(\dot{R}(\rho)S(\rho) + \dot{M}(\rho)N^T(\rho)\right)^T & A^T(\rho)S(\rho) + S(\rho)A(\rho) + \dot{S}(\rho) \end{bmatrix},$$

$$\Pi_{12}(\rho) = \begin{bmatrix} B_1(\rho) & R(\rho)C_1^T(\rho) \\ S(\rho)B_1(\rho) & C_1^T(\rho) \end{bmatrix},$$

$$\widetilde{\Omega}_f(\rho) = \begin{bmatrix} \widetilde{A}_f(\rho,\dot{\rho}) & \widetilde{B}_f(\rho) \\ \widetilde{C}_f(\rho) & D_f(\rho) \end{bmatrix},$$

$$\begin{aligned}
&\widetilde{A}_f(\rho,\dot{\rho}) = N(\rho)A_f(\rho,\dot{\rho})M^T(\rho) + N(\rho)B_f(\rho)C_2(\rho)R(\rho) + S(\rho)A(\rho)R(\rho) - (S(\rho)\dot{R}(\rho) + N(\rho)\dot{M}^T(\rho)),\\
&\widetilde{B}_f(\rho) = N(\rho)B_f(\rho),\\
&\widetilde{C}_f(\rho) = C_f(\rho)M^T(\rho) + D_f(\rho)C_2(\rho)R(\rho).
\end{aligned}$$

By the Schur Complement Lemma, (21) is then equivalent to

$$\Delta := \begin{bmatrix} \gamma I & -\widehat{D}(\rho) \\ * & \gamma I \end{bmatrix} > 0, \tag{22}$$

$$\begin{bmatrix} H_{11} + L_1\Delta^{-1}L_1^T & \left(H_{21} + L_2\Delta^{-1}L_1^T\right)^T \\ * & H_{22} + L_2\Delta^{-1}L_2^T \end{bmatrix} < 0, \tag{23}$$

where $H_{21} = A^T(\rho) + \widetilde{A}_f(\rho,\dot{\rho}) - (S(\rho)\dot{R}(\rho) + N(\rho)\dot{M}^T(\rho))$. In Algorithm 1, we solve the LMI (22) to determine feasible $D_f(\rho)$. The Eqs.(16) and (17), in which $\widetilde{B}_f(\rho)$ and $\widetilde{C}_f(\rho)$ can be solved, guarantee the diagonal blocks in (23) are negative definite [24].If there is singular condition in (16) and (17), we can achieve $\widetilde{B}_f$ and $\widetilde{C}_f$ by solving inequalities (18) and (19) directly. Set the off-diagonal block $H_{21} + L_2\Delta^{-1}L_1^T$ to zeros to compute the matrix $\widetilde{A}_f(\rho)$. According to the definition of matrices $\widetilde{A}_f(\rho), \widetilde{B}_f(\rho), \widetilde{C}_f(\rho)$, we have the gain matrices of LPV filter $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$ and $D_f(\rho)$.The transformation from (5)–(21) is equivalent, then the solved $A_f(\rho)$, $B_f(\rho)$, $C_f(\rho)$ and $D_f(\rho)$ guarantee the inequality (5) satisfied. The proof is complete. □

Remark 1. If the condition of Theorem 1 is satisfied, then the filter can always be achieved by Algorithm 1 without any extra conditions. In fact, if the inequalities (8)–(10) are satisfied, then the system (3) satisfies the Bounded Real Lemma condition (5), i.e., a filter $\Omega_f(\rho)$ exists. From the above proof, the inequalities (22) and (23) are also satisfied so that we can always obtain the matrices $D_f(\rho)$, $C_f(\rho)$, $B_f(\rho)$, $A_f(\rho)$ by Algorithm 1 based on the $R(\rho)$, $S(\rho)$ and γ. Therefore, the conservativeness (if any) comes from the condition of Theorem 1.

According to Lemma 2, the condition of Theorem 1 is equivalent to the condition of the Bounded Real Lemma which has been widely used in the existing literature. Thus, Algorithm 1 has less conservativeness.

4. Example

In this section, as an application of the proposed method in Section 3, we design a filter for a missile pitch-axis autopilot. The model taken from [1,2] is described by

$$\begin{aligned}
&\dot{\alpha}(t) = K_\alpha M(t)C_n(\alpha(t),\delta(t),M(t))\cos(\alpha(t)) + q(t),\\
&\dot{q}(t) = K_q M^2(t)C_m(\alpha(t),\delta(t),M(t)),
\end{aligned} \tag{24}$$

where the aerodynamic coefficients are

$$\begin{aligned}
&C_n(\alpha,\delta,M) = \mathrm{sgn}(\alpha)\left[a_n|\alpha|^3 + b_n|\alpha|^2 + c_n\left(2 - \frac{M}{3}\right)|\alpha|\right] + d_n\delta,\\
&C_m(\alpha,\delta,M) = \mathrm{sgn}(\alpha)\left[a_m|\alpha|^3 + b_m|\alpha|^2 + c_m\left(-7 - \frac{8M}{3}\right)|\alpha|\right] + d_m\delta,
\end{aligned}$$

and the output is normal acceleration

$$\eta(t) = K_z M^2(t)C_n(\alpha,\delta,M). \tag{25}$$

Actuator dynamic describing the tail deflection is

$$\frac{d}{dt}\begin{bmatrix} \delta(t) \\ \dot{\delta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_a^2 & -2\zeta\omega_a \end{bmatrix}\begin{bmatrix} \delta(t) \\ \dot{\delta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_a^2 \end{bmatrix}\delta_c(t). \tag{26}$$

The various variables in plant model are:

$\alpha(t)$	angle of attack (deg)
$q(t)$	pitch rate (deg/s)
$M(t)$	Mach number
$\delta_c(t)$	commanded tail deflection angle (deg)
$\delta(t)$	actual tail deflection angle (deg)
$\eta_c(t)$	commanded normal acceleration in g' s
$\eta(t)$	actual normal acceleration in g' s

Further description of various constants is provided in Table 1 and the variables $\eta(t)$ and $q(t)$ are measurable outputs.

The LPV control method in [21] is applied for the missile model above, and an ad hoc estimator for state $\alpha(t)$ is designed in [2]. The estimator is also used for switched LPV system in [3]. Substituting the ad hoc estimator in [2,3], we will apply the proposed LPV filter method in Section 3 to estimate the state $\alpha(t)$. The LPV form of missile model (24) and (25) is derived from [2] as follows:

$$\begin{bmatrix}\dot{\alpha}(t)\\ \dot{q}(t)\end{bmatrix} = \begin{bmatrix} K_\alpha \rho_2(t)\left[a_n\rho_1^2(t) + b_n|\rho_1(t)| + c_n\left(2-\frac{\rho_2(t)}{3}\right)\right]\cos(\rho_1(t)) & 1\\ K_q\rho_2^2(t)\left[a_m\rho_1^2(t) + b_m|\rho_1(t)|^2 + c_m\left(-7-\frac{8\rho_2(t)}{3}\right)\right] & 0\end{bmatrix}\begin{bmatrix}\alpha(t)\\ q(t)\end{bmatrix} + \begin{bmatrix}K_\alpha\rho_2(t)d_n\cos(\rho_1(t))\\ K_q\rho_2^2(t)d_m\end{bmatrix}\delta(t), \tag{27}$$

$$\begin{bmatrix}\eta(t)\\ q(t)\end{bmatrix} = \begin{bmatrix} K_q\rho_2^2(t)\left[a_n\rho_1^2(t) + b_n|\rho_1(t)| + c_n\left(2-\frac{\rho_2(t)}{3}\right)\right] & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\alpha(t)\\ q(t)\end{bmatrix} + \begin{bmatrix}K_z\rho_2^2(t)d_n\\ 0\end{bmatrix}\delta(t), \tag{28}$$

where $\rho_1 = \alpha$ and $\rho_2 = M$, we consider the parameter set $\mathcal{P} = [-20, 20] \times [2.4, 3.2]$. The missile system with controller and filter connection is as Fig. 2 in which the controller is as the same as in [21], and we focus on the LPV filter design.

Table 1
Coefficients in the missile system.

$K_\alpha = (0.7)\,P_0S/mv_s$	
$K_q = (0.7)\,P_0Sd/I_y$	
$K_z = (0.7)\,P_0S/m$	
$P_0 = 973.3$ lbs/ft^2	Static pressure at 20,000 ft
$S = 0.44$ ft^2	Surface area
$m = 13.98$ slugs	Mass
$v_s = 1036.4$ ft/s	Speed of sound at 20,000 ft
$d = 0.75$ ft	Diameter
$I_y = 182.5$ slug · ft^2	Pitch moment of inertia
$C_a = -0.3$	Drag coefficient
$\zeta = 0.7$	Actuator damping ratio
$\omega_a = 150$ rad/s	Actuator undamped natural frequency
$a_n = 0.000103$ deg^{-3}	
$b_n = -0.00945$ deg^{-2}	
$c_n = -0.1696$ deg^{-1}	
$d_n = -0.034$ deg^{-1}	
$a_m = 0.000215$ deg^{-3}	
$b_m = -0.0195$ deg^{-2}	
$c_m = 0.051$ deg^{-1}	
$d_m = -0.206$ deg^{-1}	

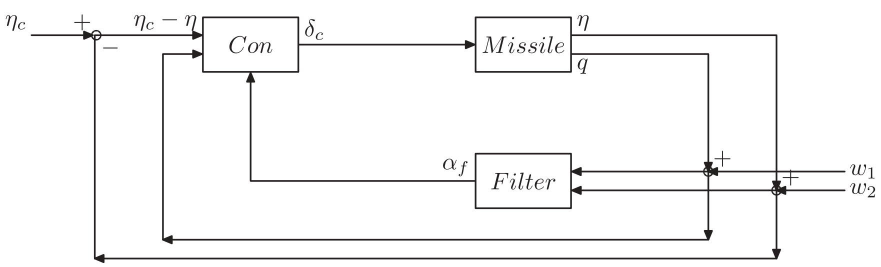

Fig. 2. Missile system with LPV controller and filter.

Choose the parameter-dependent Lyapunov function as

$$P(\rho) = P_0 + \rho_2 P_\rho, \tag{29}$$

where P_0 and P_ρ are matrices to be solved from LMI conditions in Theorem 1. Set the bound of the parameter derivative as $[-100, 100]$, and the H_∞ performance optimization result over the whole parameter region is $\gamma = 1.5962993e - 003$. Applying Algorithm 1 to the system we can achieve the LPV filter.

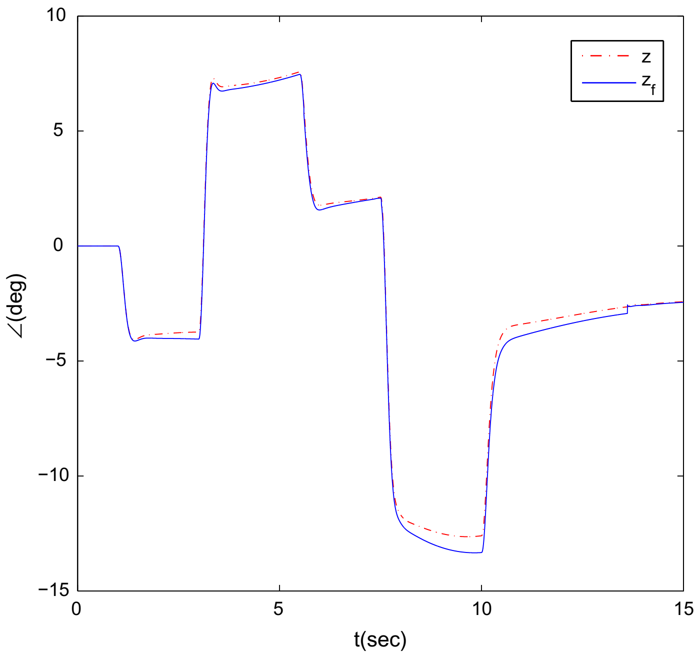

Fig. 3. The output of LPV filter and the state which is estimated.

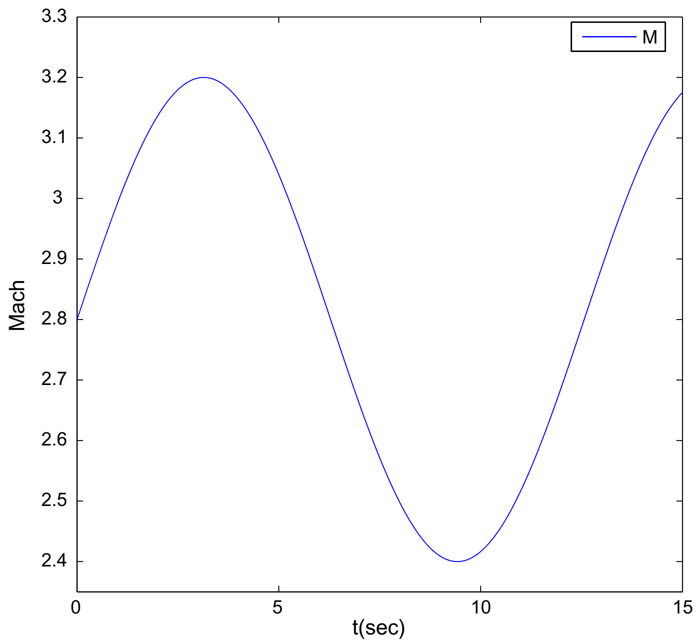

Fig. 4. The Mach number of the missile model.

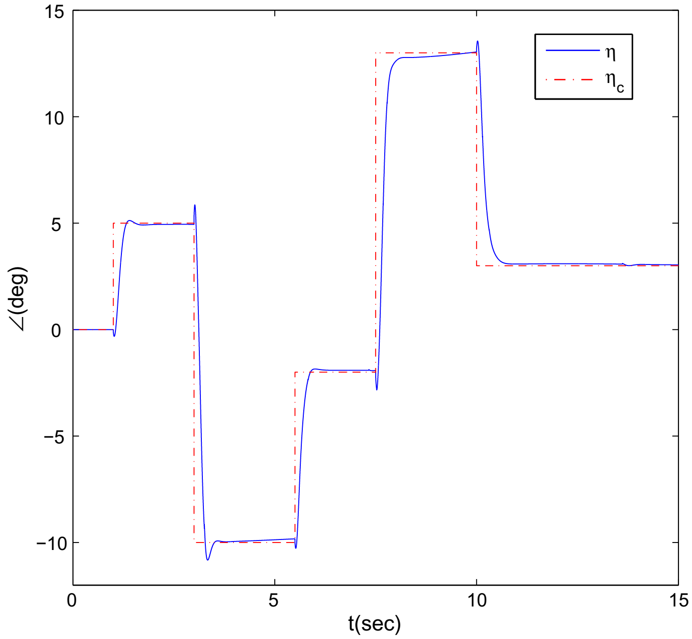

Fig. 5. The output of the LPV system and the commanded normal acceleration.

Remark 2. In the design of the filter, the Gridding method [21] is used and the γ is the max value of the H_∞ performance value in the whole parameter region. According to [1–3], the derivative boundary $[-100, 100]$ is enough for the parameter.

Simulation is performed to verify the performance of the designed filter. Since we focus on the design of the filter, the controller is designed using the method [21]. Compared with [2] we design an LPV filter while in [2] a non-LPV filter is designed. The control target of the closed-loop system is that the output η can track the command input η_c. The performance of the simulation is shown in Figs. 3–5. The filter output and the real state which is estimated are depicted in Fig. 3. The varying of parameter in the system is shown in Fig. 4. Fig. 5 shows the tracking performance of the closed-loop missile system.

5. Conclusion

This paper have presented a method to design an H_∞ filter for LPV systems with measurable parameters. For the given LPV systems, LMI conditions have been proposed for the existence of the parameter-dependent Lyapunov function and an algorithm has been proposed following the solutions to the LMI conditions. The proposed method has been demonstrated effectively through the application to the missile system.

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