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Finite-time stability analysis and controller synthesis for switched linear parameter-varying systems

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Abstract: In this paper, the finite-time stability analysis and controller synthesis for switched linear parameter-varying (LPV) systems are discussed. First based on the average dwell-time approach and convexity principle, a finite-time stability condition is presented for the switched LPV system with affine linear structured uncertainty. Second based on the derived results, the state feedback controllers and a class of switching signals with average dwell-time are designed in detail to solve finite-time stabilization problem. The main results are proved by using the piecewise parameter dependent Lyapunov function. Finally, numerical examples are given to demonstrate the effectiveness and the superiority of the proposed results.

Key Words: Switched linear parameter-varying systems, Average dwell-time, Finite-time stability, Finite-time stabilization

1 Introduction

A switched system is an important class of hybrid systems and has received a considerable attention from many researchers in the last decade. The switched system consists of several subsystems and a switching rule specifying the switches among subsystems. It can be applied into a great number of real-world systems. For example, in flight control, the controller of aircraft switches at different flight operating points along the flight trajectory [1]. (For more detail, please refer to the survey paper [2] and the references therein.)

Up to now, most of existing literatures on stability of switched systems were concentrated on Lyapunov asymptotic stability, which is defined over an infinite-time interval, such as [3-4]. However, in many realistic cases, the main concern is the behavior of the system over a fixed finite time interval, like the problem of sending a rocket from the neighborhood of a point A to the neighborhood of a point B over a fixed time interval [5]. This is probably one of reasons why the research of the finite-time stability is becoming increasingly popular. Some results on finite-time stability of switched linear system can be found in [6-7]. In [6], the concepts of finite-time stability are extended to switched linear systems. In [7], an asynchronous finite-time control problem for a class of switched linear systems is investigated. Obviously, the problems pointed out from the above literatures are focusing on the switched linear system without considering the model uncertainty (namely on the nominal switched linear system).

Since uncertainties are common in practical systems, there are many ways to describe the uncertainties of systems like polytopic structured uncertainty and affine linear structured uncertainty [8]. Though system with affine linear structured uncertainty can be transformed into a system with polytopic uncertainty, the order of the system after transformation will be increase significantly, which may result in large computational burden [9].

Motivated by the above analysis, in this paper, we address the problem of finite-time stability analysis and controller synthesis for switched linear parameter-varying (LPV) systems. Specifically, based on a piecewise parameter dependent Lyapunov function method and the average dwell-time approach, an LMI-based sufficient condition for the finite-time stability of switched linear system with affine linear structured uncertainty is derived. Then we apply the obtained method to design a set of state feedback control gains and a class of switching signals with average dwell-time to make sure that closed-loop switched LPV system is finite-time stable.

This paper is organized as follow. In the next section, the system description and some preliminaries are given. Section 3 gives the main results. First the finite-time stability analysis of switched LPV system is proposed. Second the finite-time stabilization problem for switched LPV system is dealt with. Section 4 presents two examples to illustrate the effectiveness and the superiority of the theoretical results. Finally, section 5 makes conclusion for the paper.

The notation is standard. The superscript T denotes the transpose. The symbol $(*)$ denotes symmetric blocks of symmetric matrices. $diag\{\ldots\}$ represents a block-diagonal matrix. $\lambda_{\max}(A)$ (respectively $\lambda_{\min}(A)$) represents the maximum (respectively, minimum) eigenvalue of A. The set $\{1, 2, \ldots, M\}$ is denoted by $\mathcal{M}$. The set $\{1, 2, \ldots, N\}$ is denoted by $\mathcal{N}$. If not explicitly stated, matrices are assumed to have compatible dimensions.

2 Problem formulation and preliminary

Consider the switched linear parameter-varying systems described by the following equation:

$$\dot{x} = A_{\sigma(t)}(\theta)x + B_{\sigma(t)}(\theta)u, x(0) = x_0, \qquad (1)$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m$ are the state vector, control input, respectively. $x_0 \in \mathbb{R}^n$ is the initial state value. $\theta = \left[\, \theta^{(1)}\ \theta^{(2)}\ \ldots\ \theta^{(N)} \,\right]^T \in \mathbb{R}^N$ is the parameter vector. For each $t \geqslant 0$, the switching rule $\sigma(t)$ is such that $(A_{\sigma(t)}(\theta), B_{\sigma(t)}(\theta)) \in$

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$\{(A_1(\theta), B_1(\theta)), \ldots, (A_M(\theta), B_M(\theta))\}$, where the matrices $(A_i(\theta), B_i(\theta)), i = 1, \ldots, M$ with appropriate dimensions depend affinely on the parameters $\theta^{(j)}$, i.e.:

$$A_i(\theta) = A_i^{(0)} + \theta^{(1)}A_i^{(1)} + \cdots + \theta^{(N)}A_i^{(N)},$$
$$B_i(\theta) = B_i^{(0)} + \theta^{(1)}B_i^{(1)} + \cdots + \theta^{(N)}B_i^{(N)}.$$

Each element $\theta^{(j)}$ in parameter vector θ ranges between known extreme minimum and maximum values, i.e.: $\underline{\theta}^{(j)} \leqslant \theta^{(j)} \leqslant \overline{\theta}^{(j)}, j \in \mathcal{N}$, which means that the parameter vector θ takes value in a polytope Ω, where

$$\Omega := \left\{\theta = \left[\theta^{(1)}\theta^{(2)}\ldots\theta^{(N)}\right]^T : \underline{\theta}^{(j)} \leqslant \theta^{(j)} \leqslant \overline{\theta}^{(j)}, j \in \mathcal{N}\right\}.$$

In the sequel,

$$\mathcal{V} := \left\{v = \left[v^{(1)}v^{(2)}\ldots v^{(N)}\right]^T : v^{(j)} \in \{\underline{\theta}^{(j)}, \bar{\theta}^{(j)}\}, j \in \mathcal{N}\right\}$$

denotes the set of 2^N vertices or corners of this parameter box Ω. In addition, rewrite the form of the set $\mathcal{V}$ as $\mathcal{V} = \{v_1, v_2, \ldots, v_{2^N}\}$, where vector $v_k \in \mathbb{R}^N, \forall k \in \{1, 2, \ldots, 2^N\}$.

Next, we assume the time derivative of parameter vector $\dot{\theta}$ is bounded and satisfies $\underline{\omega}^{(j)} \leqslant \dot{\theta}^{(j)} \leqslant \overline{\omega}^{(j)}, j \in \mathcal{N}$. That is $\dot{\theta}$ belongs to a polytope Λ and the corresponding vertex set of Λ is defined as:

$$\mathcal{W} := \{\omega = \left[\,\omega^{(1)} \;\; \omega^{(2)} \;\; \ldots \;\; \omega^{(N)}\,\right]^T : \omega^{(j)} \in \{\underline{\omega}^{(j)}, \bar{\omega}^{(j)}\}.$$

Next, we introduce some conceptions about finite-time stable for switched linear systems.

Definition 1. [6] (Finite-time stability)
Given three positive constants c_1, c_2, T, with $c_1 < c_2$, a positive definite matrix R and a given switching signal $\sigma(t) \in \{1, \ldots, M\}$, the switched LPV system (1a) with $u \equiv 0$ is said to be finite-time stable with respect to (c_1, c_2, T, R, σ), if $x_0^T R x_0 \leqslant c_1 \Rightarrow x(t)^T R x(t) < c_2, \forall t \in (0, T]$.

Definition 2. [10] (Average dwell-time)
For any switching signal $\sigma(t)$ and any $T > t$, let $N_\sigma(t, T)$ denote the switching numbers of $\sigma(t)$ over the interval $[t, T)$. For the given $\tau_a > 0$ and an integer $N_0 \geqslant 0$, if the inequality $N_\sigma(t, T) \leqslant N_0 + \frac{T-t}{\tau_a}$ holds, then the positive constant τ_a is called an average dwell-time and N_0 is called a chattering bound.

As is commonly used in the literature, for convenience, we choose $N_0 = 0$ in this paper.

The aim of this paper is, based on the average dwell-time method and convexity principle, to find sufficient conditions for the finite-time stability of uncertain switched system. Then design a set of state feedback controller gains K_i for ensuring the finite-time stability of system (1) with respect to (c_1, c_2, T, R, σ).

Before getting the main results in this paper, the following basic lemma is introduced.

Lemma 1. [11]
Consider a scalar quadratic function of $\theta \in \mathbb{R}^N$,

$$f(\theta) = \alpha_0 + \sum_i \alpha_i\theta^{(i)} + \sum_{i<j} \beta_{ij}\theta^{(i)}\theta^{(j)} + \sum_i \gamma_i(\theta^{(i)})^2$$

and assume that $f(\cdot)$ is multiconvex, that is $2\gamma_i = \frac{\partial^2 f}{\partial \theta_i^2}(\theta) \geqslant 0$ for $i = 1, \ldots, N$. Then $f(\cdot)$ is negative for all θ in the hyper-rectangle Ω if and only if $f(v) < 0$ for all v in the vertex set $\mathcal{V}$.

Lemma 2. [12]
Let A be a symmetric matrix and B, C be matrices of appropriate dimensions. The following statements are equivalent:

(1)
$$A < 0, \quad A + BC^T + CB^T < 0$$

(2) The LMI problem

$$\begin{bmatrix} A & B + CF \\ * & -F - F^T \end{bmatrix} < 0$$

is feasible with respect to F.

3 Main results

In this section, firstly, some results will be presented regarding the finite-time stability of the switched LPV system.

Consider a multiple parameter-dependent Lyapunov function of the form

$$\begin{aligned} P_i(\theta) &= P_i^{(0)} + \left[\, P_i^{(1)} \;\ldots\; P_i^{(N)} \,\right]\theta \\ &= P_i^{(0)} + \sum_{j=1}^{N} \theta^{(j)} P_i^{(j)}, \end{aligned} \tag{2}$$

where $\theta \in \Omega$ and $P_i^{(0)}, \ldots, P_i^{(j)}$ are symmetric matrices for $i \in \mathcal{M}$, $j \in \mathcal{N}$. Then based on this kind of Lyapunov function, the following Lemma are proposed.

Lemma 3. For any $i \in \mathcal{M}$, denote $\tilde{P}_i(\theta) = R^{\frac{1}{2}} P_i(\theta) R^{\frac{1}{2}}$, where $P_i(\theta)$ is of the form (2). If there exist symmetric matrices $P_i^{(0)}, \ldots, P_i^{(N)}$ and constant $\alpha \geqslant 0$ satisfying , for all $i \in \mathcal{M}$, the following inequalities:

$$P_i(\theta) = P_i^{(0)} + \sum_{j=1}^{N} \theta^j P_i^{(j)} > 0, \tag{3}$$

$$A_i(\theta)^T \tilde{P}_i(\theta) + \tilde{P}_i(\theta) A_i(\theta) + \dot{\tilde{P}}_i(\theta) - \alpha \tilde{P}_i(\theta) < 0, \tag{4}$$

$$\mu < \frac{c_2}{c_1} e^{-\alpha T}, \tag{5}$$

then the switched LPV system (1) with $u \equiv 0$ is finite-time stable with respect to (c_1, c_2, T, R, σ) for any switching signal σ with average dwell-time such that

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln(c_2/c_1) - \ln \mu - \alpha T}, \tag{6}$$

where $\lambda_1 = \min\limits_{\forall i \in \mathcal{M}, \theta \in \Omega} (\lambda_{\min}(P_i(\theta)))$, $\lambda_2 = \max\limits_{\forall i \in \mathcal{M}, \theta \in \Omega} (\lambda_{\max}(P_i(\theta)))$ and $\mu = \frac{\lambda_2}{\lambda_1}$.

Proof. Take a piecewise parameter-dependent Lyapunov function as follows

$$V(x(t)) = V_{\sigma(t)}(x(t)) = x(t)^T \tilde{P}_{\sigma(t)}(\theta) x(t).$$

Then following the same line as in the proof of Theorem 2 in [6], we can get the above Lemma. □

Though Lemma 3 has introduced a simple sufficient condition for the stability of switched LPV system, the inequalities in (4-5) are difficult to solve directly. Therefore, Based on the Lemma 3, we derive sufficient conditions for the stability of switched LPV system in the LMIs form which are more efficient for calculation.

Theorem 1. If there exist symmetric matrices $P_i^{(0)},\dots,P_i^{(N)}$, constant $\alpha \geqslant 0$, matrices Z_i and a scalar κ which verifies $P_i(\omega)-(2\kappa+\alpha)P_i(v)-P_i^{(0)}<0$ satisfying the following inequalities:

$$P_i(v)=P_i^{(0)}+\sum_{j=1}^{N}v^{(j)}P_i^{(j)}>0, \tag{7}$$

$$\begin{bmatrix} P_i(\omega)-(2\kappa+\alpha)P_i(v)-P_i^{(0)} & * \\ P_i(v)+Z_i^TA_i(v)+\kappa Z_i^T & -Z_i-Z_i^T \end{bmatrix}<0, \tag{8}$$

$$\frac{c_1}{c_2}e^{\alpha T}I<P_i(v)<I, \tag{9}$$

for all $i\in\mathcal{M}$ and $(v,\omega)\in\mathcal{V}\times\mathcal{W}$, where

$$P_i(\omega)=P_i^{(0)}+\sum_{j=1}^{N}\omega^{(j)}P_i^{(j)},$$

$$A_i(v)=A_i^{(0)}+\sum_{j=1}^{N}v^{(j)}A_i^{(j)}.$$

Then the switched LPV system (1) with $u\equiv 0$ is finite-time stable with respect to (c_1,c_2,T,R,σ) for any switching signal σ with average dwell-time such that

$$\tau_a>\tau_a^*=\frac{T\ln\mu}{\ln(c_2/c_1)-\ln\mu-\alpha T}, \tag{10}$$

where $\lambda_1=\min\limits_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(P_i(v)))$, $\lambda_2=\max\limits_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(P_i(v)))$, $\mu=\frac{\lambda_2}{\lambda_1}$.

Proof. Step 1: By applying the Lemma 1, it is easily to verify that the inequality (7) is equivalent to the (3) in Lemma 3, which implies that (3) holds if matrix inequality (7) is satisfied.

Step 2: By using the Lemma 1, the inequality (8) is equivalent to

$$\begin{bmatrix} P_i(\dot\theta)-(2\kappa+\alpha)P_i(\theta)-P_i^0 & * \\ P_i(\theta)+Z_i^TA_i(\theta)+\kappa Z_i^T & -Z_i-Z_i^T \end{bmatrix}<0, \quad \forall\theta\in\Omega,\dot\theta\in\Lambda, \tag{11}$$

where $P_i(\dot\theta)=P_i^{(0)}+\sum\limits_{j=1}^{N}\dot\theta^{(j)}P_i^{(j)}$,Through some simple algebraic operations, we obtain that the inequality (11) is equivalent to

$$\begin{bmatrix} \dot P_i(\theta)-2\kappa P_i(\theta)-\alpha P_i(\theta) & * \\ P_i(\theta)+Z_i^TA_i(\theta)+\kappa Z_i^T & -Z_i-Z_i^T \end{bmatrix}<0, \quad \forall\theta\in\Omega. \tag{12}$$

On the other hand, due to the definition of κ, it implies that $\dot P_i(\theta)-2\kappa P_i(\theta)-\alpha P_i(\theta)<0$. Next, according to the Lemma 2, the inequality (12) is equivalent to

$$\begin{aligned}(A_i^T(\theta)+\kappa I)P_i(\theta) &+ P_i(\theta)(A_i(\theta)+\kappa I)\\ &+ \dot P_i(\theta)-2\kappa P_i(\theta)-\alpha P_i(\theta)<0.\end{aligned}$$

The above inequality can be rewritten as

$$A_i^T(\theta)P_i(\theta)+P_i(\theta)A_i(\theta)+\dot P_i(\theta)-\alpha P_i(\theta)<0. \tag{13}$$

Through Pre- and post-multiplying the inequality (13) by the known matrix $R^{\frac{1}{2}}$, it obviously shows that inequality (8) is equivalent to the inequality (4) in Lemma 3, which implies that (8) is the sufficient condition for (4).

Step 3: The condition (9) implies that $\lambda_{\max}(P_i(v))<1$ and $\frac{c_1}{c_2}e^{\alpha T}I<\lambda_{\min}(P_i(v))$ hold for all $i\in\mathcal{M}$, $v\in\mathcal{V}$. Due to the definition of μ in Theorem 1, one can obtain

$$\mu=\frac{\max\limits_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(P_i(v)))}{\min\limits_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(P_i(v)))}<\frac{c_2}{c_1}e^{-\alpha T}. \tag{14}$$

Hence, if we can prove

$$\max_{\forall i\in\mathcal{M},\theta\in\Omega}(\lambda_{\max}(P_i(\theta)))=\max_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(P_i(v))), \tag{15}$$

$$\min_{\forall i\in\mathcal{M},\theta\in\Omega}(\lambda_{\min}(P_i(\theta)))=\min_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(P_i(v))), \tag{16}$$

then we will obtain that condition (9) is the sufficient condition for (5) in Lemma 3. First since $v\in\mathcal{V}\subset\Omega$, we can easily obtain

$$\max_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(P_i(v)))\leqslant\max_{\forall i\in\mathcal{M},\theta\in\Omega}(\lambda_{\max}(P_i(\theta))), \tag{17}$$

$$\min_{\forall i\in\mathcal{M},\theta\in\Omega}(\lambda_{\min}(P_i(\theta)))\leqslant\min_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(P_i(v))). \tag{18}$$

Considering the set $\mathcal{V}$ is the vertices set of the convex polytope Ω, based on the convexity principle, we obtain $\theta=\sum\limits_{k=1}^{2^N}\beta_kv_k$, where v_k is defined previously, $\beta_k\geqslant 0,\forall k\in\{1,2,\dots,2^N\}$ and $\sum\limits_{k=1}^{2^N}\beta_k=1$. Next, the following equation can be obtained:

$$\begin{aligned}
P_i(\theta) &= P_i(\sum_{k=1}^{2^N}\beta_kv_k)\\
&= P_i^{(0)}+\begin{bmatrix}P_i^{(1)} & P_i^{(2)} & \cdots & P_i^{(N)}\end{bmatrix}(\sum_{k=1}^{2^N}\beta_kv_k)\\
&= P_i^{(0)}+\sum_{k=1}^{2^N}\beta_k\begin{bmatrix}P_i^{(1)} & P_i^{(2)} & \cdots & P_i^{(N)}\end{bmatrix}v_k\\
&= \sum_{k=1}^{2^N}\beta_kP_i^{(0)}+\sum_{k=1}^{2^N}\beta_k\begin{bmatrix}P_i^{(1)} & P_i^{(2)} & \cdots & P_i^{(N)}\end{bmatrix}v_k\\
&= \sum_{k=1}^{2^N}\beta_k(P_i^{(0)}+\begin{bmatrix}P_i^{(1)} & P_i^{(2)} & \cdots & P_i^{(N)}\end{bmatrix}v_k)\\
&= \sum_{k=1}^{2^N}\beta_kP_i(v_k).
\end{aligned} \tag{19}$$

Next, using the equation (19), we have

$$\begin{aligned}\lambda_{\max}(P_i(\theta)) &= \lambda_{\max}(\sum_{k=1}^{2^N}\beta_k P_i(v_k)) \\ &\leqslant \lambda_{\max}(\sum_{k=1}^{2^N}\beta_k \lambda_{\max}(P_i(v_k))I) \\ &= \sum_{k=1}^{2^N}\beta_k \lambda_{\max}(P_i(v_k)) \\ &\leqslant \sum_{k=1}^{2^N}\beta_k \max_{v\in\mathcal{V}}\{\lambda_{\max}(P_i(v))\}, \\ &= \max_{v\in\mathcal{V}}\{\lambda_{\max}(P_i(v))\}. \end{aligned} \tag{20}$$

From the inequality (20), we obtain

$$\max_{\forall i\in\mathcal{M},\theta\in\Omega}(\lambda_{\max}(P_i(\theta))) \leqslant \max_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(P_i(v))). \tag{21}$$

By combining (17) and (21), the equation (15) is obtained. Then taking the same method, from (19), we can obtain

$$\begin{aligned}\lambda_{\min}(P_i(\theta)) &= \lambda_{\min}(\sum_{k=1}^{2^N}\beta_k P_i(v_k)) \\ &\geqslant \lambda_{\min}(\sum_{k=1}^{2^N}\beta_k \lambda_{\min}(P_i(v_k))I) \\ &= \sum_{k=1}^{2^N}\beta_k \lambda_{\min}(P_i(v_k)) \\ &\geqslant \sum_{k=1}^{2^N}\beta_k \min_{v\in\mathcal{V}}\{\lambda_{\min}(P_i(v))\}, \\ &= \min_{v\in\mathcal{V}}\{\lambda_{\min}(P_i(v))\}, \end{aligned} \tag{22}$$

and the inequality

$$\min_{\forall i\in\mathcal{M},\theta\in\Omega}(\lambda_{\min}(P_i(\theta))) \geqslant \min_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(P_i(v))). \tag{23}$$

Finally, by combining (18) and (23), the equation (16) is obtained. As a result, the condition (9) is the sufficient condition of (5).

Step 4: It follows from the (15) and (16) that the (10) is equivalent to (6) in Lemma 3.

Thus the proof of Theorem 1 is complete. □

Remark 1. Theorem 1 presents sufficient conditions for finite-time stability of switched LPV system which helps to deal with the following stabilization problem. In terms of the piecewise Lyapunov function chose in [6], the multiple parameter-dependent Lyapunov function method adopted here has lower conservatism.

Based on the above analysis, next, the following theorem gives a sufficient condition for the finite-time stabilization of the switched LPV system. For simple and convenience, we develop the following Corollary and then the state feedback controllers are investigated.

Corollary 1. If there exist symmetric matrices $Q_i^{(0)}, \ldots, Q_i^{(N)}$, constant $\alpha \geqslant 0$, matrices W_i and a scalar κ which verifies $Q_i(\omega) - (2\kappa+\alpha)Q_i(v) - Q_i^{(0)} < 0$ satisfying the following inequalities:

$$Q_i(v) = Q_i^{(0)} + \sum_{j=1}^{N} v^{(j)} Q_i^{(j)} > 0, \tag{24}$$

$$\begin{bmatrix} Q_i(\omega) - (2\kappa+\alpha)Q_i(v) - Q_i^{(0)} & * \\ Q_i(v) + A_i(v)W_i + \kappa W_i & -W_i - W_i^T \end{bmatrix} < 0, \tag{25}$$

$$W_i + W_i^T - I > Q_i(v), \tag{26}$$

$$\begin{bmatrix} -Q_i(v) & W_i^T \\ W_i & -\frac{c_2}{c_1}e^{-\alpha T} I \end{bmatrix} < 0, \tag{27}$$

for all $i \in \mathcal{M}$ and $(v,\omega) \in \mathcal{V}\times\mathcal{W}$, where

$$Q_i(\omega) = Q_i^{(0)} + \sum_{j=1}^{N} \omega^{(j)} Q_i^{(j)},$$

$$A_i(v) = A_i^{(0)} + \sum_{j=1}^{N} v^{(j)} A_i^{(j)}.$$

Then the switched LPV system (1) with $u \equiv 0$ is finite-time stable with respect to (c_1, c_2, T, R, σ) for any switching signal σ with average dwell-time such that

$$\tau_a > \tau_a^* = \frac{T\ln\mu}{\ln(c_2/c_1) - \ln\mu - \alpha T}, \tag{28}$$

where $\lambda_1 = \min_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(W_i^{-T}Q_i(v)W_i^{-1}))$, $\lambda_2 = \max_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(W_i^{-T}Q_i(v)W_i^{-1}))$, $\mu = \frac{\lambda_2}{\lambda_1}$.

Proof. Let $W_i = Z_i^{-1}$. Pre- and post-multiplying the inequalities (7) and (8) by $diag\{W_i^T, W_i^T\}$ and its transpose respectively. Defining $Q_i^{(0)} = W_i^T P_0 W_i$, $Q_i(v) = W_i^T P_i(v) W_i$, $Q_i(\omega) = W_i^T P_i(\omega) W_i$, the conditions (24) and (25) in Corollary 1 are obtained.

Noting

$$(W_i - I)^T I (W_i - I) \geqslant 0,$$

we have

$$W_i^T W_i \geqslant W_i + W_i^T - I. \tag{29}$$

By using (29) and Shur complement Lemma, the (26) and (27) which presented as the form of LMIs lead to the following inequality:

$$\frac{c_1}{c_2}e^{\alpha T} I < W_i^{-T}Q_i(v)W_i^{-1} < I. \tag{30}$$

Considering the definition of $Q_i(v)$, the (30) is equivalent to (9), which implies that (26) and (27) are the sufficient conditions for (9).

At last, based on the definition of $Q_i(v)$, the following equations are obtained:

$$\min_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(P_i(v))) = \min_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\min}(W_i^{-T}Q_i(v)W_i^{-1})), \tag{31}$$

$$\max_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(P_i(v))) = \max_{\forall i\in\mathcal{M},v\in\mathcal{V}}(\lambda_{\max}(W_i^{-T}Q_i(v)W_i^{-1})). \tag{32}$$

By combining (28) with (31) and (32), the (28) can be expressed as the condition (10) in Theorem 1 equivalently, which completes the proof. □

Applying the approach presented, it is easy to obtain an LMI design method for the finite-time stabilization by state-feedback controllers as follows.

Theorem 2. If there exist symmetric matrices $Q_i^{(0)}, \ldots, Q_i^{(N)}$, constant $\alpha \geqslant 0$, matrices Y_i, W_i and a scalar κ which verifies $Q_i(\omega) - (2\kappa+\alpha)Q_i(v) - Q_i^{(0)} < 0$ satisfying the following inequalities:

$$Q_i(v) = Q_i^{(0)} + \sum_{j=1}^{N} v^j Q_i^{(j)} > 0, \tag{33}$$

$$\begin{bmatrix} Q_i(\omega) - (2\kappa+\alpha)Q_i(v) - Q_i^{(0)} & * \\ Q_i(v) + A_i(v)W_i + B_i(v)Y_i + \kappa W_i & -W_i - W_i^T \end{bmatrix} < 0, \tag{34}$$

$$W_i + W_i^T - I > Q_i(v), \tag{35}$$

$$\begin{bmatrix} -Q_i(v) & W_i^T \\ W_i & -\frac{c_2}{c1} e^{-\alpha T} I \end{bmatrix} < 0, \tag{36}$$

for all $i \in \mathcal{M}$ and $(v, \omega) \in \mathcal{V} \times \mathcal{W}$, where

$$Q_i(\omega) = Q_i^{(0)} + \sum_{j=1}^{N} \omega^{(j)} Q_i^{(j)},$$

$$A_i(v) = A_i^{(0)} + \sum_{j=1}^{N} v^{(j)} A_i^{(j)},$$

$$B_i(v) = B_i^{(0)} + \sum_{j=1}^{N} v^{(j)} B_i^{(j)}.$$

Then under the state-feedback gain computed as $K_i = Y_i W_i^{-1}$, the closed-loop switched LPV system is finite-time stable with respect to (c_1, c_2, T, R, σ) for any switching signal σ with average dwell-time such that

$$\tau_a > \tau_a^* = \frac{T \ln \mu}{\ln(c_2/c_1) - \ln \mu - \alpha T}, \tag{37}$$

where $\lambda_1 = \min_{\forall i \in \mathcal{M}, v \in \mathcal{V}} (\lambda_{\min}(W_i^{-T} Q_i(v) W_i^{-1}))$, $\lambda_2 = \max_{\forall i \in \mathcal{M}, v \in \mathcal{V}} (\lambda_{\max}(W_i^{-T} Q_i(v) W_i^{-1}))$, $\mu = \frac{\lambda_2}{\lambda_1}$.

Proof. By Substituting state-feedback controllers $u(t) = K_{\sigma(t)} x(t)$ into system (1), the closed-loop system is described as follows

$$\dot{x} = (A_{\sigma(t)}(\theta) + B_{\sigma(t)}(\theta) K_{\sigma(t)}) x, x(0) = x_0. \tag{38}$$

Then applying Corollary 1 for closed loop system (23) and letting $Y_i = K_i W_i$, we can obtain the Theorem 2. □

Remark 2. Theorem 2 gives us a method to design state feedback controllers and a class of switching signals with average dwell-time to dealing with the finite-time stabilization of an uncertain switched system. Noticing that κ is a positive scalar that parameterizes the LMI constraints. It gives an additional degree of freedom for designing the state feedback gains.

4 Examples

In this section, for illustrating the effectiveness and the superiority of the proposed method, numerical examples are presented as follows. Moreover, Example 1 shows the effectiveness of the controller synthesis method given by Theorem 2. By compared with the controller synthesis method given in Theorem 4 in [6], Example 2 hopes to illustrate that the proposed method can achieve better performance when we consider the model uncertainty.

4.1 Example 1:

We consider the switched LVP system (1) with

$$A_1(\theta) = A_1^{(0)} + \theta A_1^{(1)} = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix} + \theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_2(\theta) = A_2^{(0)} + \theta A_2^{(1)} = \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} + \theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_1(\theta) = B_1^{(0)} + \theta B_1^{(1)} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + \theta \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.3 \end{bmatrix},$$

$$B_2(\theta) = B_2^{(0)} + \theta B_2^{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \theta \begin{bmatrix} 0 & 0 \\ 0.2 & 0 \end{bmatrix},$$

where the time-varying scalar $\theta(t)$ is defined as $\theta(t) \in [0, 1]$ and let its time derivative $\dot{\theta}(t) \in [-0.5, 0.5]$. According to the limitation of $\theta(t)$ given above, here, we choose $\theta(t) = 0.5\sin(t) + 0.5$. Let $c_1 = 1$, $c_2 = 20$, $T = 5$, $R = I$, $\alpha = 0.01$, $\kappa = 20$ and $x_0 = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}^T$, by Theorem 2, a feasible solution can be obtained as

$$Q_1^{(0)} = \begin{bmatrix} 3.2398 & 0.2935 \\ 0.2935 & 2.8823 \end{bmatrix}, Q_1^{(1)} = \begin{bmatrix} 0.3091 & -0.1343 \\ -0.1343 & 1.0340 \end{bmatrix},$$

$$Q_2^{(0)} = \begin{bmatrix} 6.1164 & -0.1889 \\ -0.1889 & 6.3112 \end{bmatrix}, Q_2^{(1)} = \begin{bmatrix} -0.1643 & 0.4196 \\ 0.4196 & -0.2764 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 3.9551 & 0.1919 \\ 0.1747 & 3.9889 \end{bmatrix}, W_2 = \begin{bmatrix} 6.5762 & 0.0087 \\ 0.0199 & 6.6729 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -79.3014 & 3.2288 \\ 61.9098 & -79.4531 \end{bmatrix}, Y_2 = \begin{bmatrix} -138.426 & 12.8799 \\ 1.8275 & -142.557 \end{bmatrix}.$$

The state feedback gains can be obtained as

$$K_1 = \begin{bmatrix} -20.1288 & 1.7780 \\ 16.5682 & -20.7159 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -21.0556 & 1.9575 \\ 0.3427 & -21.3641 \end{bmatrix}.$$

According to the (37), we can get $\tau_a > \tau_a^* = 1.41$. Therefore, under the state feedback controllers $u = K_1 x$, $u = K_2 x$ and for any switching signal $\sigma(t)$ with average dwell-time $\tau_a > \tau_a^* = 1.41$, the closed loop switched LPV system is finite-time stable with respect to $(1, 20, 5, I, \sigma(t))$. For this example, the states response of the switched LPV system under a periodic switching signal with interval time $\Delta T = 1.5s$ is shown in Fig. 1. Fig. 1(a) and Fig. 1(b) show the state trajectories and the quadratic performance index $x^T R x$ over $0 \sim 5s$ under the switching law $\sigma(t)$, which is shown in Fig. 1(c). From Fig. 1, it is concluded that the given uncertain system is finite-time stable with respect to $(1, 20, 5, I, \sigma(t))$ under the state feedback controllers $u = K_1 x$, $u = K_2 x$ and switching signal $\sigma(t)$.

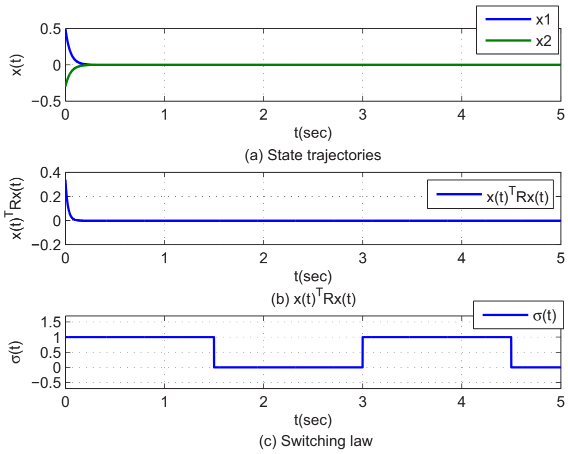

Fig. 1: The system response in Example 1

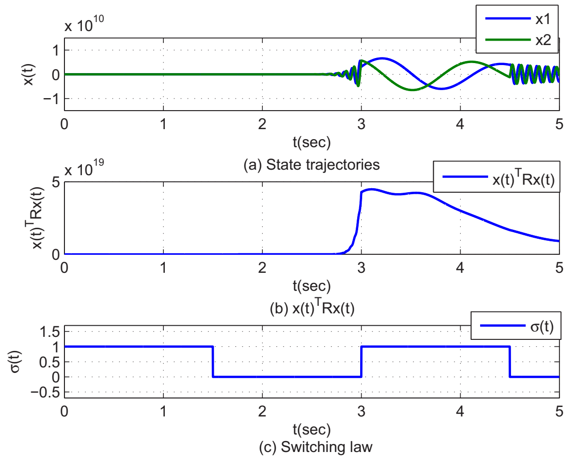

Fig. 2: The system response in Example 2

4.2 Example 2:

For illustrating the superiority of the method presented in this paper, we consider another method to solve the finite-time stabilization problem. With the approach given in Theorem 4 in [6], we can design state feedback controllers for the nominal switched LPV system:

$$\dot{x}(t) = A^{(0)}_{\sigma(t)} x(t) + B^{(0)}_{\sigma(t)} u$$

Using the same given conditions from above, the state feedback gains can be obtained as

$$K_1^* = \begin{bmatrix} -0.7021 & -69.0913 \\ 68.7934 & 68.3892 \end{bmatrix},$$
$$K_2^* = \begin{bmatrix} -0.7021 & 7.1047 \\ -6.1047 & -0.7021 \end{bmatrix}.$$

Applying the same switching signal $\sigma(t)$ which have defined in Example 1, we conduct the simulation of the close-loop switched LPV system with state feedback gains K_1^* and K_2^*. The results are present in Fig. 2. Similarly, Fig. 2(a) and Fig. 2(b) show the state trajectories and the quadratic performance index $x^T Rx$ over $0 \sim 5s$ under the switching law $\sigma(t)$, which is shown in Fig. 2(c). In contrast to Fig. 1, it can be clearly seen that the given system is not finite-time stable due to the time-varying parameter θ. Thus, it is necessary for practical applications to take the model uncertainty into account and the method presented in this paper can deal with the finite-time stabilization problem of the switched LPV system.

5 Conclusion

In this paper, first we have proposed the finite-time stability conditions for the switched LPV system with affine linear structured uncertainty. Second we have studied the problem of designing state feedback controllers and a class of switching signals with average dwell-time to dealing with the finite-time stabilization of an uncertain switched system. Finally, numerical examples have been shown to demonstrate the effectiveness and the superiority of the proposed results.

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